

# Strehl Ratio Split for Production—Limited Optics

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## Abstract

A Strehl Ratio (SR) decomposition for diffraction limited optics is proposed leading to a criterion of  $SR \geq 97\%$  for perfect—production optics.

What is beyond diffraction limited optics? Well, nothing if we consider the strict definition<sup>1</sup> or a pinhole camera. Both systems are free of aberrations. From a more practical point of view diffraction limited optics is often characterized by the relative size of tolerable aberrations. Qualitatively, the point spread by transverse aberrations should be comparable or even smaller than the Airy disk resulting from diffraction. Quantitatively, there are two traditional numbers in use: The Rayleigh limit for the peak to valley (PV) value of the OPD ( $PV \leq \lambda/4$ ) and the Marechal criterion for the variance of the wavefront deformation ( $RMS \leq \lambda/4$ ). Both are adjusted to a Strehl ratio (SR) of  $SR \geq 80\%$ , which serves for the common meaning of diffraction limited.

The remaining 20% beyond this limit call for further attention nowadays. Concerning fast high-end optics they span a variety of quality classes. A modern microscope objective, *e.g.*, is monochromatically and on-axis designed to about  $OPD \leq \lambda/100$ , even for numerical apertures as high as 0.95. This is far beyond diffraction limited and on the border of physical limits. However, the manufactured quality of such objectives is obviously restricted by the production process. Concerning the final quality this is the only remaining degree of freedom to be optimized. Therefore, from a designers point of view and related to the spec of *e.g.* on-axis performance these objectives might be called production limited. While a perfect design objective undoubtedly is defined by  $SR = 100\%$ , it is not realistic to claim the same for a perfect-production objective, at least not for a series production.

Depending on the application of high-end optics, even on-axis and monochromatically, a description of manufactured objectives only by their SR is not sufficient. On the other hand the Zernike coefficients obtained by interferometric measurements often cover more information than useful. In contact with our customers from the semiconductor business we found it convenient to decompose the SR into three factors. Beside it's handiness, our proposed procedure can be founded on theoretical and experimental reasons.

The SR is related to the RMS-value of the wavefront aberrations by

$$SR \approx e^{-(2\pi RMS)^2} \quad (1)$$

This approximation is well suited for the considered region of  $SR \geq 80\%$  and the deviation is less than 1%. For an unvignetted pupil the RMS value can be calculated by the coefficients of the well known Zernike expansion of the wavefront

$$W(p, \theta) = \sum_{n=0}^{\infty} \sum_{m=0}^n \sqrt{\frac{2n+2}{1+\delta_{nm}}} \times R_n^m(p) \left( c_{nm} \cos(m\theta) + s_{nm} \sin(m\theta) \right) \quad (2)$$

Applying the numbering and normalizing scheme of Mahajan<sup>2</sup> this expression may be rewritten in the form

$$W(p, \theta) = \sum_{j \in \mathbb{N}} a_j Z_j(p, \theta) \quad (3)$$

where the  $Z_j$  are normalized to unity. The variance of the aberration function is then given by

$$RMS^2 = \sum_{j \geq 2} a_j^2 \quad (4)$$

According to a symmetry of the Zernike Polynomials  $Z_j(p, \theta)$  this sum can canonically be divided into three parts: those contributions without an angle dependence, and those with an odd or even angle frequency. We stress that, deviating from Mahajan, the terms odd or even are not related to the sin or cos terms, but to the multiplicity of the angle dependence, *i.e.*, connected to odd and even values of  $m$  in Equation 2. For obvious but non-necessary reasons we call these contributions spherical (SPH), coma (COM) and astigmatism (AST) arrive at:

$$RMS^2 = RMS_{SPH}^2 + RMS_{COM}^2 + RMS_{AST}^2 \quad (5)$$

Up to the fourth order, *e.g.*, according to the numbering of Mahajan

$$RMS_{SPH}^2 = a_4^2 + a_{11}^2$$

$$RMS_{COM}^2 = a_2^2 + a_3^2 + a_7^2 + a_8^2 + a_9^2 + a_{10}^2 \quad (6)$$

$$RMS_{AST}^2 = a_5^2 + a_6^2 + a_{12}^2 + a_{13}^2 + a_{14}^2 + a_{15}^2 \quad (7)$$

From the RMS decomposition the split of the SR follows as

$$SR = SR_{SPH} \times SR_{COM} \times SR_{AST} \quad (8)$$

This expression is meaningful on-axis as well as off-axis as long as vignetting can be neglected. A more puristic treatment would include only the terms  $m=1$ , instead of  $m$  odd, and  $m=2$ , instead of  $m$  even, in the above definition of the SR-factors called coma and astigmatism. An extra factor would then be necessary to account for the rest. We avoided this procedure for simplicity reasons. Additionally, from our experience these terms are very small. Lastly, the proposed split works very well with the compensation scheme described below.

For rotational symmetric systems there is only one possible on-axis monochromatic aberration (SPH) as considered during the design process. Nevertheless, production adds two more: on-axis coma due to decentrations and on-axis astigmatism mainly caused by cylindric surface errors. All three types can be and are minimized independently: the spherical part by an adjustable airspace, coma by a radial shiftable lens group and astigmatism by rotating different lens groups relative to each other. From practical experience we learned that a compensation efficiency of 99% for each type is achievable, even under the conditions of series production. This in turn will result in an overall SR of

$$SR \geq 97\% \quad (9)$$

which is a reasonable value to agree on for a perfect-production objective in series production. Obviously, this definition is restricted to high-end diffraction limited systems. To demonstrate that we do not propose pure desktop-numbers, the result of a recent assembled batch of 1003 /0.90 microscope objectives is displayed in Figure 1. An impressive part, as we feel it, fulfils the required spec<sup>9</sup>. Therefore, this spec is not only a claim, but practically confirmed as a suitable criteria for perfect-production optics. Whether the proposed criterion will be time dependent, is a question of future demands and engineer-

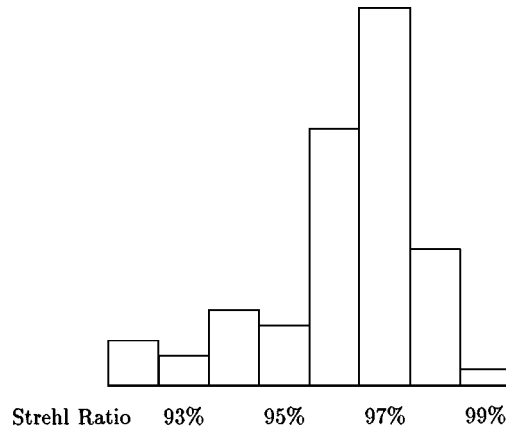


FIGURE 1. The relative distribution of Strehl ratios for a recent batch of 1113 /0.90 microscope objectives.

ing developments.

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#### References

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2. V.N. Mahajan, "Zernike circle polynomials and optical aberrations of systems with circular pupils," Eng. Lab. Notes in Opt. Phot. News, 5 (1994).