Unaided Visual Acuity and Blur: A Simple Model

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Abstract

Purpose. This article proposes a simple model that describes the quantitative relationship between unaided visual acuity and blur attributed to refractive errors.

Methods. The standard model for describing the relationship between visual acuity and blur, as published by Raasch, is used as a starting point to develop a simpler model based on heuristic arguments. The basis of Raasch's data is augmented by published findings in the range of low-level refractive errors. Sphero-cylindrical refractive errors are transformed into a single blur quantity b, also termed dioptric distance, which serves as an input in both models. The possible influence of the cylinder axis and the pupil size is not included. Results. The quite simple model for the unaided minimum angle of resolution, MARp$\propto$ $1 + b^2$, nicely matches available data and improves the SE of the regression by a factor of 2 in comparison to Raasch's model.

Conclusions. Both models considered in this article describe measurement data equally well. They differ in terms of complexity and functional form. The simple model provides a valid description for low-level refractive errors, where Raasch's model fails. Actual uncertainties in experimental data on unaided visual acuity, especially the frequent lack of information on pupil diameter, prevent meaningful numerical comparison and the refinement of both models. However, theoretical arguments are provided in support of the simple model.

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Key Words: visual acuity, refractive error, blur, dioptric distance, minimum angle of resolution, sphero-cylindrical refractive error

Visual acuity is a frequently used indicator of spatial vision that is applied both in clinical settings and for legal matters like driver's licenses. The impact of refractive errors on visual acuity has been of interest for a long time. Here, we discuss standard, daylight, unaided visual acuity for monocular, central vision. Because of the many factors influencing the relationship between visual acuity and refractive errors, it is quite a challenge to provide a quantitative description of this relationship. It mainly depends on the type and quantity of the refractive error in the uncorrected eye, for example, spherical and cylindrical refractive errors and aberrations of higher order, whereas pupil size plays an important role in all cases.[1] We will confine ourselves to the impact of ordinary refractive errors, sphere, and astigmatism and neglect higher-order aberrations. The effect of the cylinder axis on visual acuity will, for the most part, be neglected. Furthermore, measurement data on uncorrected visual acuities often suffer from incomplete testing protocols, missing data on pupil size, illumination levels, target types, and so on. An overview of all these topics can be found in the lucid review article by Smith[2] and the textbook by Bennett and Rabbetts.[3] Nevertheless, a very simple equation to describe unaided visual acuity as a function of the refractive error will be presented in this article. Visual acuity is measured in different ways. One possible procedure is the free Freiburg acuity test using randomized optotypes in the form of Landolt rings.[4] Although many parameters are involved in the relationship between visual acuity and refractive error, Raasch, in a seminal paper,[5] was able to demonstrate two important points. First, there is a useful fit to empirical data and, second, a single combination of sphero-cylindrical refraction data into a single scalar blur quantity leads to a working approach. Although Raasch's equation generally works well and has come to be recognized as a kind of standard, it has one drawback: it does not allow for small defocus or astigmatism values and becomes undefined for an emmetropic eye with a refractive error of zero. Nevertheless, Raasch's equation is repeatedly used for small blur values.[6, 7] The slight deficiency in Raasch's formula becomes important when the influence of refraction errors must be estimated, say, for a merit function in the design process for progressive addition lenses or contact lenses. Additionally, tolerance specifications on oph- thalmic devices often require at least an estimate of the impact on visual acuity. Manufacturers of spectacle lenses often use

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in-house formulas for these purposes. Their equations, however, are rarely published, an exception being the work of Fauquier et al.[8] This article proposes a quite simple equation that describes the influence of refractive errors on uncorrected visual acuity. To this end, we introduce Raasch's approach and the question of how blur can be quantified.

Raasch's Approach

From a given sphero-cylindrical refractive error of a subject, where S and C denote sphere and cylinder respectively, a scalar blur quantity, b, measured in diopters, can be calculated: either with a Pythagorean addition of the equivalent sphere and a cross cylinder of strength $\pm C/2$ or, equivalently, as the mean of the quadratic curvatures in the principal meridians. This leads to

$$b^2 = (S + C/2)^2 + \left(\frac{C}{2}\right)^2 = \frac{1}{2}\left(S^2 + (S + C)^2\right) \qquad (1)$$

The square root of this quantity, which is always positive or zero, is called vector length in Raasch's paper. Dioptric distance is another term in use. Some readers might be familiar with power vectors described in terms of M, J_0, and J_{45}. Equation 1 could also be written as $b^2 = M^2 + J_0^2 + J_{45}^2$. It should be noted that the blur quantity does not change when recipe values are transposed for a different cylinder sign. Nor does it depend on the cylinder axis, assuming that all meridians produce the same effects on visual acuity. The dependence of visual acuity on the cylinder axis may or may not be true; the question is currently being examined. In the case of a pure spherical power, $C = 0$, we simply have $b = |S|$. In the case of hypermetropia and a working accommodation, the quantity b might be misleading, because partial compensation by the accommodating eye lens is not automatically included. Patients with a cycloplegia or fully developed presbyopia are allowed for, because no accommodation takes place. If, however, accommodation totally compensates for a refractive error, we have $b = 0$. Optical arguments can be derived from the use of power matrices or power vectors in favor of the above form of the blur quantity.[9, 10] When the visual acuity V is defined as the inverse of the minimum angle of resolution (MAR) and expressed as decimal acuity, Raasch's equation establishes a relationship between the logarithm of the MAR and the logarithm of the blur quantity $x = \log(b)$:

$$-\log V = \log(\text{MAR}) = a_0 + a_1 x + a_2 x^2 \qquad (2)$$

The three coefficients of the polynomial in the variable x are $a_0 = 0.48$, $a_1 = 1.07$ and $a_2 = 0.46$. All logarithms are used to the base 10. From Fig. 2 of Raasch's paper it appears that empirical data are abundant in the range 1.5 diopters (D) $< b <$ 9D, whereas the $0.5 < b < 1.5$ range is sparsely populated. Nearly no data seem to be used in the range $b < 0.4$ D. Because of the logarithmic form of the regression variable, $x = \log(b)$, the value of b must be bounded from below, say $b > 0.1$ D, to prevent an ill-defined situation. Because Raasch used little or no data for small values of b in that regression, this restriction does not seem to be essential. In conclusion, Raasch's approach explains 92% of the variance in acuity scores underlying his regression. A certain degree of caution is required in interpreting this result, be-cause of the large range of the underlying data.[11] Nevertheless, this regression model will be the starting point and reference for a simple model, which will be introduced at this stage.

A Simple Model

The approach proposed here uses the same blur quantity as defined by Raasch. Therefore, no change is made to the input form of sphero-cylindrical data. These data are converted to the blur quantity by means of equation 1. However, the functional form for describing visual acuity will be altered in two ways: First, we will use a relative unaided visual acuity, defined as the fraction

$$V_{\text{rel}} = \frac{V}{V_{bc}} \qquad (3)$$

where the unaided visual acuity V is divided by the visual acuity related to the best correction, V_{bc} (see Refs. [8] and [12] for similar approaches). Instead of the notion of a relative variable, we also use the term *normalized variable*. The use of a blur quantity implicitly assumes knowledge of the recipe values leading to the best correction. Whether the visual acuity corresponding to the best correction is available is another subject altogether. The use of a normalized variable V_{rel} does not imply $V_{bc} = 1$, but $V_{rel} = 1$ for $b = 0$. The introduction of a relative variable might reduce the influence of some experimental uncertainties, because they affect both quantities: the numerator and the denominator. Not only experimental uncertainties, but also certain clinical conditions, for example, cataract, retinal pathologies, age-related effects, and so on, would influence both visual acuities in a similar fashion. However, because the model is based on data from otherwise normal eyes, the model probably applies best to normal eyes. Furthermore, relative variables do not have a unit: in other words, they are dimensionless. Therefore, they do not depend on the chosen unit of length, angle, Snellen fraction, and so on. The inverse of V_{rel} is MAR_{rel} and the only difference between the logarithms of the two quantities is a minus sign. Second, instead of the variable x given above, we directly apply the square of the blur quantity b in the following extremely simple functional form:

$$V_{\text{rel}} = \frac{1}{1 + b^2} \qquad \text{or} \qquad \text{MAR}_{\text{rel}} = 1 + b^2 \qquad (4)$$

As a simple example we consider a pure defocus ($C = 0$). We get, say for the numerical values of $b = S = \{1; 2; 3\}$ DS, the relative visual acuities of $V_{\text{rel}} = \{0.5; 0.2; 0.1\}$. Because the logarithm of 0.5 is -0.3, we have a loss of 3 lines for one diopter of spherical power. Relative visual acuity in this case means a loss of 3 lines independent of the line, which corresponds to the best correction.

Equation 4 is the central proposal in this article and the equation will now be tested against Raasch's model and against further empirical data as described in the next section.

Empirical data and results

To augment Raasch's data for small blur values, we include further data. We make use of data from Holladay et al.[13] for spherical power values in the interval from 0.5 DS up to 5 DS. In a similar range, we extracted data from Atchison et al.[1] The data provided by Villegas et al.[14] lie in a smaller interval, $b \leq 2$ D. The data from Ohlendorf et al.[12] represent astigmatic blur, either induced as a cylindrical error or as a cross-cylinder. They span the range $b \leq 2.25$ D. Fauquier et al.[8] supplied data for 50 sphero-cylindrical combinations and fall in the region $b < 1.5$ D. Watanabe et al.[15] provide data for astigmatic blur from 0.5 DC in steps of 0.50 DC up to 2.50 DC. Finally, the data of Kamiya et al.[16] describe the effect of artificial pupil sizes from 1 to 5 mm on the unaided visual acuity while astigmatism of 1, 2, and 3 DC is induced. All data are pooled together and the 82 items are binned in increasing order in intervals of 0.25 D according to the blur quantity b. Only data for pupil diameters in the range from 2 to 5 mm were included. For each bin, the mean (logarithmic) and the SE are calculated. Fig. 1 shows these data together with the simple model and Raasch's model. The error bars for measurement data represent the SEs owing to all kind of variations, including pupil size as the dominant factor.

Regarding typical experimental uncertainties, there is clearly no difference between Raasch's model and the simple model presented here for the range $5 \leq b \leq 10$ D. Therefore, for large values of the blur quantity, both approaches render the same results. Because of the mathematical difficulties in Raasch's formula for $b \to 0$, it is difficult to normalize his results, and for that reason, we took the formula as it is. A constant offset of -0.13 log units would be introduced if the value for $b = 0.1$ D were applied as a reference for normalization. In the interval $1\text{D} < b < 5$ D, results in Raasch's model are consistently lower than those of the simple model.

However, for small blur values, the functional forms are quite different. Because the simple model assumes maximum relative visual acuity, the slope of the function decreases toward zero, until vanishing for the emmetropic eye.

The simple model appears to be free of any parameters. Actually, it contains at least one implicit parameter: the coefficient of b^2, which has a physical unit of square meters and a numerical value of 1. We would like to know whether a numerical value different from 1 would be consistent with the data. To this end, we introduce an explicit parameter a, which accounts for a possible deviation from the numerical value of 1. Therefore, we choose the form $(1 + a)$ as the coefficient of b^2. Hence, the claim $a = 0$ defines the simple model and can be tested by fitting the parameter a to the data. The fit could be done by a linear regression. Because of heteroscedastic residuals, we instead applied a nonlinear regression for $\log V_{rel}$, which yields an estimate close to zero, $a = 0.02$. As a result, the null hypothesis, $a = 0$, cannot be rejected with a p value of 0.70. In other words, the parameter a is very unlikely a meaningful addition to the simple model. The confidence interval (2.5% to 97.5%) of the parameter a is given by (-0.09 to 0.14) and the R^2 (adj.) value for the nonlinear fit is 0.99. A more appropriate number than R^2 in the case of a nonlinear regression is the SE of the regression. It takes a value of 0.046 when the simple model is applied to the data considered in the current article. Raasch's model, applied without any changes to the same data, renders a value of 0.11. In this case, the simple model shows an improvement by a factor of 2 for the SE of the regression.

This section may thus be summarized as follows: in view of experimental uncertainties, an amazingly simple model with a minimal number of parameters offers a sufficient "primal sketch" for the relationship between unaided visual acuity and refractive error.

DISCUSSION

Although the best-corrected visual acuity is remarkably stable over the range of natural daylight pupil diameters, say 2 to 4 mm, the unaided visual acuity depends heavily on pupil size in the case of blurred images. The explanation for the former goes back to the Stiles-Crawford effect, as given by Vohnsen.[17] The latter phenomenon is well known and can be demonstrated by a simple test. When a pinhole is placed in front of an ametropic eye, visual acuity can be increased drastically even for considerable refractive errors if the cause of ametropia lies in the optical pathway. With arguments from geometrical optics, this fact can be explained by the reduction of the blur circle area, which is proportional to the area of the pupil - drastically reduced by a small pinhole. How- ever, the maximal visual acuity achieved with a pinhole is bounded by diffraction effects and clearly falls short of the visual acuity rendered by the best correction with natural pupil diameters.

When the eye suffers from an astigmatic refractive error, the wavefront reaching the retina renders a blur ellipse instead of a blur circle. The area of this blur ellipse might be a proxy for the blurring effect, which reduces visual acuity. Clearly, the area of a cross section in the Sturm conoid degenerates to zero when the cross section contains the tangential or sagittal focus. This means that the area of the ellipse is useful only far from the regions of focus. In these distant regions, where the sphere is large compared with the cylinder, the area of the ellipse is proportional to the product $|S(S + C)|$, where S and $S + C$ are the principal curvatures of the wavefront. The square of the blur quantity can likewise be approximated by $b^2 \approx |S(S+C)|$. The unaided visual acuity actually decreases at a rate of $V_{rel} \propto 1/b^2$. Thus, asymptotically, the unaided visual acuity is inversely proportional to the area of the blur ellipse.

This asymptotic behavior naturally emerges from the simple model (equation 4). It is difficult to see how such a behavior could be derived from equation 2, although it implicitly describes the same effect. For small sphere and cylinder values, or for large cylinder values, this simple model breaks down, requiring replacement of the blur ellipse area with a more sophisticated quantity, as given by the blur quantity b.

The argument that the increase in the area, rather than the linear dimension, is responsible for decreasing visual acuity is quite unusual. This approach might be supported by the fact that the number of photons decreases in proportion to the area over which they are spread, leading to a lower signal-to-noise ratio. However, this argument is quite speculative and calls for further discussion that is beyond the scope of this article.

The best correction should maximize visual acuity. Up until now, at least, there has been widespread agreement on this point, although maximization of a different merit quantity, like contrast at intermediate spatial frequencies, could be a goal as well. Nevertheless, every maximum has the property that small variations in parameters (like refractive error) have no effect - at least in a linear approximation. The variations manifest themselves only in quadratic order. In other words, the tangent to an extremal point is horizontal and the slope vanishes at the maximum. The simple model shows this property. For small values of b, we have: $V_{rel} \approx 1 - b^2$ and $\log(V_{rel}) \approx -b^2$. No linear term in b is present, as expected for an extremum. The relative visual acuity (or its logarithm) decreases quadratically as it approaches its best value. This might appear to be quite a gradual decline. When depth of focus ($\gtrapprox 0.25$ D) is considered, there is actually only a slow response to defocus. From the simple model, we have a drop-off of 3 lines (0.3 log units) at a refractive error of 1 D, which contrasts with the rule of thumb of "4 lines per diopter." Again, in the absence of sufficient information on pupil diameter, this difference is not significant. Before us, Smith[2] suggested a similar approach to the behavior of low-level refractive errors. It is worth mentioning that when expanded to quadratic order, the formula proposed by Smith leads to the same numerical result when his proposal $k = 0.8$ and a pupil diameter of 2.5 mm are used. Smith dismissed any functional form of the type $\log V - \log b$, because it has no foundation in optical theory.

The simple model presented here can be augmented by introducing parameters like pupil size, axis orientation of a cylinder, or linear dependence on b. However, experimental data show substantial uncertainties, and results from different authors are not compatible, because experimental parameters are not known, not documented, or not standardized. As long as these circumstances prevail, the simplest approach, which does not contradict experimental findings, should be appropriate. This article does not argue that the simple model describes data better than Raasch's does; it merely states that owing to limited information on pupil diameter or to experimental uncertainties, the two models deliver similar results. This suggests that a simpler model would have much to recommend it.

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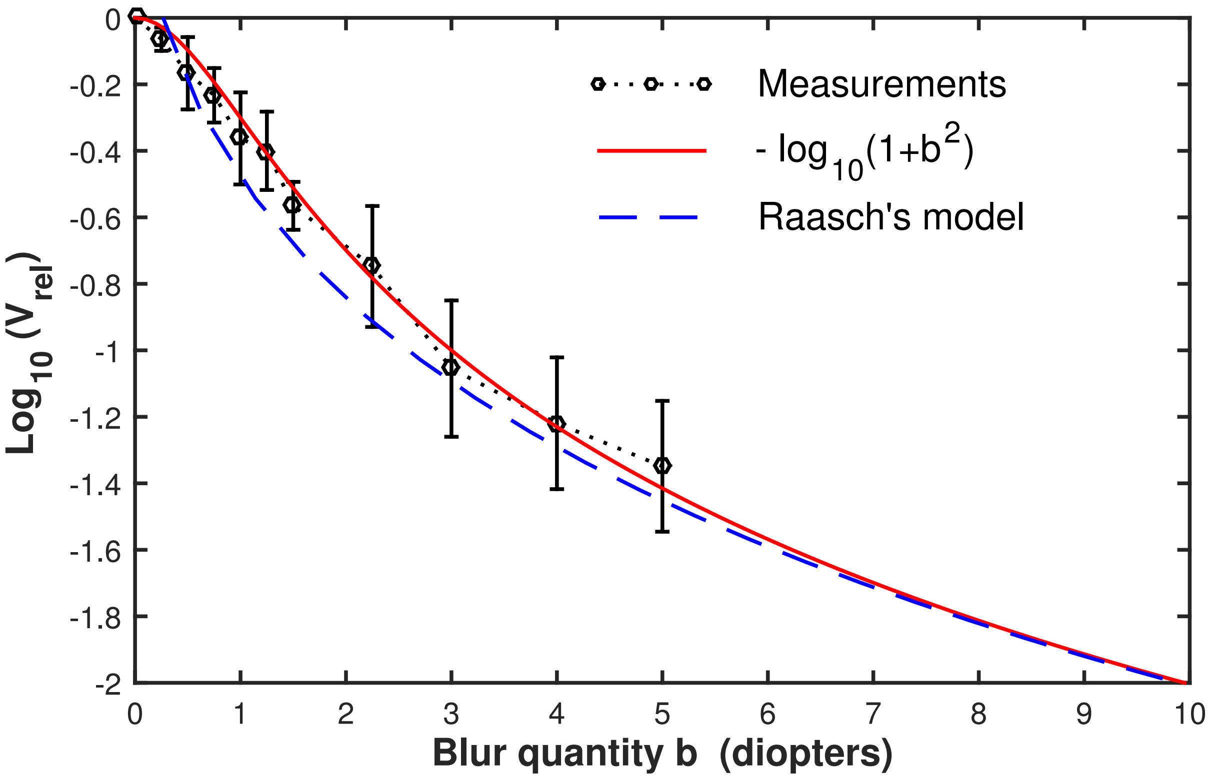

FIGURE 1.

The logarithm of relative unaided visual acuity (negative of logMAR) is plotted against the blur variable b. Measurement data in the range $b \leq 5$ D have been pooled from different studies and binned into 0.25-D intervals. Only data for pupil sizes between 2 and 5 mm are included. In each interval, the circle stands for the mean (logarithmic) and the error bar represents the SE. The variance is mainly dominated by the pupil size. The model of Raasch (broken line without marks) and the simple model $(-\log(1+b^2))$ (solid line without marks) show an overall agreement with experimental data. Both models share the same asymptotic behavior for large values of b. For $b > 3$ D, the difference between the two models is less than 0.1 log units and therefore negligible. The SE of the regression is improved by a factor of 2 for the simple model.