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From α-decay to exotic decays – a unified model*

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The possibility of extending the microscopic α-decay theory to exotic decay modes is investigated. It is argued that the strong Pauli distortions in the antisymmetrized wave function of the open channel favour this possibility. The microscopic description can be reduced to a one-body picture; the structure information is then contained in a spectroscopic factor. Existing microscopic calculations from α- up to ^{16}O-decays justify a simple unified bulk formula for the spectroscopic factor. In this way a semiempirical formula for the decay constant is obtained depending on 1 parameter only (for odd and even nuclei, respectively). This formula describes all decay modes in a unified way and yields an excellent reproduction of the known exotic decay rates. It is thus well-suited for the unambiguous prediction of yet unmeasured decay constants.

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1. Introduction

1.1. Alpha decay

Experimentally one finds that a 1 MeV difference in the α-energy E_α changes the life time of an α-unstable nucleus by about 4 orders of magnitude; this relation is described by the Geiger-Nuttall law. One of the most prominent successes of quantum mechanics was the explanation of the α-decay and the Geiger-Nuttall law by Gamov [1] and Condon and Gurney [2]. This explanation is given in a one-body model: the parent nucleus is represented by an α-particle moving in the average potential $V(R_\alpha)$ of the daughter nucleus. The α-particle can leave the attractive interior region of $V(R_\alpha)$ only by penetrating through the Coulomb barrier. The penetration probability and thus the related Gamov decay constant λ_G depend sensitively on the released energy E_α.

In the energy regime of interest, the nuclei are well described as many-body systems made up by nucleons. The most successful and most widely applicable many-body wave functions are those of the nuclear shell model [3]. This shell model is also the basic starting point for most of the more sophisticated microscopic descriptions of nuclear structure. For the first time Mang [4] evaluated the α-decay constant using microscopic shell model wave functions. The calculation is performed within the following space of many-body states

$$\{\phi_{A+a}, \mathscr{A}(u_E(\mathbf{R}_{aA})\,\phi_a\,\phi_A)\} \tag{1}$$

where ϕ_{A+a}, ϕ_A and ϕ_a are the many-body wave functions of the parent nucleus, the daughter nucleus and the α-particle, respectively. The function $u_E(\mathbf{R}_{aA})$ describes the relative motion in the open channel, and $\mathscr{A}$ denotes the antisymmetrization operator.

Mang's calculations [5] were able to describe the relative decay rates successfully. The absolute decay rates, however, fell short to the experimental ones by a factor of the order 10^2 to 10^3. This problem was solved [6] by noting the following effect: The antisymmetrization between ϕ_a and ϕ_A leads to a normalization defect. Therefore the amplitude u_E in (1) contains a normalization part. This part has to be split off from u_E in order to obtain a wave function of relative motion which can be determined by a one-body Schrödinger equation. Consequently this normalization appears in the spectroscopic factor. The decay constants obtained by proper consideration of this effect are of the right order of magnitude. Alternative attempts to solve the problem of absolute decay rates by extensive configuration mixing are discussed in [7].

1.2. Extension to exotic decays

The discovery of the ^{14}C-decay by Rose and Jones [8] in 1984 initiated an extensive search for other exotic decays. Already measured are the emission of C, Ne,

* Dedicated to Prof. Dr. H.J. Mang on the occasion of his 60th birthday

Mg and Si nuclei. A review of the experimental status up to 1989 is given in [9, 10], for the latest results see [11–13, 26]. Quantitative arguments for the emission of fragments heavier than α-particles have been given for the first time by Sandulescu et al. in 1980 [14].

We want to discuss the possibility of extending the successful α-decay theory to these exotic decay modes. In such an extended model the wave function ϕ_a in (1) stands for the light fragment (for example ^{16}O, $a=16$). The physical basis is represented by the configuration space (1) which contains the wave function of the quasi-bound parent nucleus (mass number $A+a$, proton number $Z+z$) and the antisymmetrized product state for the open channel (fragments (A, Z) and (a, z)).

The main argument against an extension of (1) to exotic decays is the following. The open channel in (1) is made up by the *undisturbed* states ϕ_a and ϕ_A of the two fragments (usually ground states). One might argue that this is only justified for a *sudden* process. For discussing this argument we denote the typical time scale for the motion of the small fragment by t_a (for example the orbiting time of an α inside the nucleus), and a typical time scale of ϕ_A by t_A (for example a time period of a collective oscillation or deformation). The use of the undisturbed wave functions ϕ_a and ϕ_A is then assumed to be justified if $t_a \ll t_A$. Since the time scales are related to the corresponding masses the condition might be met for α-decay (because of $a=4 \ll A$) but, eventually, would be violated for exotic decays. This argument against the extension of (1) to exotic decays is, however, inconclusive for the following reason: Due to the antisymmetrization the open channel state in (1) contains strong *Pauli distortions* and is actually far away from a sudden configuration. This will be explained in more detail in the following paragraphs.

In order to discuss the Pauli distortions explicitly we consider an α-particle and a closed shell daughter nucleus as an example. The internal structure of the α-particle is described by an oscillator ground state ϕ_α. For its motion we use a Gaussian wave packet u_0 centered at $\mathbf{R}$ in coordinate and at $\hbar\mathbf{K}$ in momentum space:

$$u_0(\mathbf{R}_\alpha)\,\phi_\alpha = c\exp(\mathrm{i}\mathbf{K}\mathbf{R}_\alpha)\exp(-\gamma(\mathbf{R}_\alpha-\mathbf{R})^2)\,\phi_\alpha$$
$$= \prod_{j=1}^{4}\exp(\mathrm{i}\mathbf{K}\mathbf{r}_j/4)\,\varphi_{1s}(\mathbf{r}_j-\mathbf{R}) := (1s')^4. \qquad (2)$$

The centre-of-mass coordinate of the α-particle is denoted by $\mathbf{R}_\alpha$, the nucleon coordinates by $\mathbf{r}_j$. The spin and isospin part ($S=0$, $T=0$) is suppressed. For a suitable value of γ the wave function (2) can be written as a product of four $1s$-states $\varphi_{1s}(\mathbf{r}) \propto \exp(-\beta(\mathbf{r}-\mathbf{R})^2)$ centered at $\mathbf{R}$; due to the phase factor these $1s$-states are moving with the average momentum $\hbar\mathbf{K}/4$. Accordingly, for a fragment with a nucleons the average nucleon momentum becomes $\hbar\mathbf{K}/a$. The dependence on $\mathbf{R}$ and $\mathbf{K}$ is denoted by a prime in the shorthand notation $1s'$.

In a closed shell configuration ϕ_A the lowest single particle states $\nu_1, \nu_2, \ldots, \nu_A$ are occupied. With (2) the wave function $u_0\,\phi_\alpha\,\phi_A$ becomes a product of $A+a$ single particle states. The A states in ϕ_A are mutually orthogonal, but not orthogonal to those of the α-particle. In a Slater determinant built of $A+a$ nucleon states the non-orthogonal parts cancel. Therefore the antisymmetrization leads effectively to the following replacement:

$$|1s'\rangle \xrightarrow{\text{Pauli distortion}} |\widetilde{1s'}\rangle = |1s'\rangle - \sum_{j=1}^{A}|\nu_j\rangle\langle\nu_j|1s'\rangle$$
$$= \sum_{j=A+1}^{\infty}|\nu_j\rangle\langle\nu_j|1s'\rangle. \qquad (3)$$

This means that the nucleons of the α-particle are effectively restricted to the levels outside the closed shell (last expression). The argument can be easily extended to ^{16}O instead of an α-particle. For non-closed shells (for ϕ_a or ϕ_A) some technical problems in performing the antisymmetrization arise but the basic physical effects are the same.

Equation (3) displays the Pauli distortions induced by the antisymmetrization. These distortions are quite strong: Placing the α-particle at rest within the nuclear surface of a lead nucleus every nucleon of the α-particle finds its place to about 90% $\left(\sum_1^A|\langle\nu_j|1s'\rangle|^2 \simeq 0.9\right)$ occupied, which is equivalent to the percentage of the momenta of $1s'$ which are below the Fermi momentum $\hbar k_F$. This number implies that the resulting configuration is far from being sudden.

The inclusion of an average momentum $\mathbf{K}$ in the above formula allows an estimate of the momenta necessary for a sudden configuration. The overlaps $\langle\nu_j|1s'\rangle$ in (3) are small if the condition $K/4 \gg k_F$ is fulfilled. For a light fragment with a nucleons this means

$$K \gg a k_F \Rightarrow \text{sudden case: } \rho = \rho_a + \rho_A. \qquad (4)$$

If this condition is fullfilled the Pauli distortions are small and the nucleon density ρ of the combined system is $\rho_a+\rho_A$. Instead, for $K \lesssim a k_F$, only a moderate enhancement of the nuclear density [15] occurs in the overlap region (instead of a doubling).

The fragment ϕ_a is formed by the last a single particle states in ϕ_{A+a}. Therefore its relevant momenta are of the order $a\hbar k_F$ and the theory based on (1) does not correspond to the sudden limit. Quantitatively we may describe the strength of the Pauli distortions by the normalization defect of $|\widetilde{1s'}\rangle$ in (3). For normalized states $|1s'\rangle$ one obtains $\langle\widetilde{1s'}|\widetilde{1s'}\rangle \simeq 0.1$ for an α-particle at rest, and about 0.2 to 0.3 for $K \approx 4k_F$ if $\mathbf{R}$ is in the region of the nuclear surface. This discussion applies to α- and exotic decays in the same way.

Summarizing, we state that the open channel description in (1) is far from being a sudden description. In the antisymmetrized open channel state the nucleons of ϕ_a are automatically pushed above the Fermi level of ϕ_A. Thereby a configuration is obtained which in contrast to the sudden limit ($\rho=\rho_a+\rho_A$) is 'dynamically acceptable'. By this we mean that there is no density doubling in the overlap region and that therefore additional

dynamical effects are not overwhelming. In principle, dynamical effects could be taken into account by including further (open or closed) channels in (1).

We conclude this subsection with the statement that it is not unplausible to extent the α-decay theory to exotic decays. We proceed accordingly and present a unified model applicable to α-decay *and* to exotic decay modes.

2. Unified model

2.1. Reduction to the one-body level

On the one-body level (corresponding to [1, 2]) the decay constant λ_G is determined by the solution of the Schrödinger equation,

$$\left(-\frac{\hbar^2}{2\mu}\Delta + V(R) - E\right)\varphi_E(\mathbf{R}) = 0 \quad \Rightarrow \lambda_G. \tag{5}$$

Here μ is the reduced mass of the $A+a$-channel and $V(R)$ is a suitable potential for this channel. The one-body decay constant $\lambda_G = \Gamma/\hbar$ is given by the width Γ of the (very sharp) resonance of the scattering solution of (5) at the appropriate energy E_a. Alternatively one may consider a time dependent Schrödinger equation with the initial condition that the wave function is localized in the interior. The probability of finding the particle in the interior behaves then like $\exp(-\lambda_G t)$.

In the parent nucleus one finds the structure of the open channel (the antisymmetrized (ϕ_A plus ϕ_a)-structure) with a certain probability S. To the extent to which this structure is preformed in the parent nucleus the one-body model (5) applies. This means that the actual decay constant λ becomes

$$\lambda = \lambda_G S. \tag{6}$$

For a more elaborate treatment of the reduction to the one-body level we refer to [4, 6]; this treatment uses reduced amplitudes instead of their norm S. The inherent errors due to uncertainties in the potential V and the structure of the involved nuclei are considerable (see Sects. 2.2 and 2.3); this is in particular so for the exotic decays. Therefore the simple formulation (6) is quite appropriate.

The probability of finding the open channel structure in the parent nucleus ϕ_{A+a} equals the expectation value of the projection operator $\hat{P}$ onto this structure:

$$S = \langle \phi_{A+a} | \hat{P} | \phi_{A+a} \rangle. \tag{7}$$

The structure of the open channel is defined by the continuous set of the basis states $|\mathbf{R}\rangle$:

$$\langle \mathbf{r}_1, \ldots, \mathbf{r}_{A+a} | \mathbf{R} \rangle = \mathscr{A}(\delta(\mathbf{R} - \mathbf{R}_a)\, \phi_a \phi_A). \tag{8}$$

The daughter nucleus wave function $\phi_A(\mathbf{r}_1, \ldots, \mathbf{r}_A)$ depends on A nucleon coordinates; this means that the centre-of-mass motion is neither split off from the heavy fragment nor from the parent nucleus. The light fragment wave function ϕ_a depends on $a-1$ internal coordinates only. Due to the Pauli distortions the states $|\mathbf{R}\rangle$ are not normalized to $\delta(\mathbf{R}-\mathbf{R}')$ but to

$$\langle \mathbf{R} | \mathbf{R}' \rangle = (1 - \hat{K})_{\mathbf{R},\mathbf{R}'} = \delta(\mathbf{R} - \mathbf{R}') - K(\mathbf{R}, \mathbf{R}'). \tag{9}$$

The projection operator $\hat{P}$ onto the space $\{|\mathbf{R}\rangle\}$ is then

$$\hat{P} = \int \mathrm{d}\mathbf{R} \int \mathrm{d}\mathbf{R}' |\mathbf{R}\rangle \left(\frac{1}{1-\hat{K}}\right)_{\mathbf{R},\mathbf{R}'} \langle \mathbf{R}'|. \tag{10}$$

The spectroscopic factor S defined in this way equals the percentage to which the parent nucleus state lies in the space spanned by (8); it is thus the probability of finding the open channel structure preformed in the parent nucleus.

2.2. Spectroscopic factor

We discuss the evaluation of the resulting spectroscopic factor (7) and its dependence on the nucleon number a.

The norm operator $(1-\hat{K})^{-1}$ in (10) is needed for the property $\hat{P}^2 = \hat{P}$. This projection property is required for defining a proper quantum mechanical probability (7). Cluster spectroscopic factors found in the literature before 1975 are those which result from the neglect of the norm operator in (10); in contrast to S we call them conventional spectroscopic factor S_{conv}. The replacement of S_{conv} by S solves the problem of absolute values (as discussed in the last paragraph of Sect. 1.1).

Microscopic calculations of S have been presented for α-decay in [6, 17] and for ^{14}C-decay in [18]. Including additional calculations up to ^{16}O we display some typical values:

$$S \simeq \begin{cases} 10^{-2} & {}^{212}\mathrm{Po} \to \alpha + {}^{208}\mathrm{Pb} \\ 10^{-8} & {}^{222}\mathrm{Ra} \to {}^{12}\mathrm{C} + {}^{210}\mathrm{Pb} \\ 10^{-10} & {}^{224}\mathrm{Ra} \to {}^{14}\mathrm{C} + {}^{210}\mathrm{Pb} \\ 10^{-11} & {}^{226}\mathrm{Th} \to {}^{16}\mathrm{O} + {}^{210}\mathrm{Pb} \end{cases} \tag{11}$$

The corresponding conventional spectroscopic factors are smaller by factors of the order 300, 10^6, 10^8 and 10^{12}, respectively. Nevertheless, S_{conv} has been used for exotic decays in [16].

The spectroscopic factor is related to the overlaps of the single particle states of ϕ_a with the last a states in ϕ_{A+a}. In the simplified picture underlying (2) the light fragment is represented by the single particle states $|\mu_1\rangle, \ldots, |\mu_a\rangle$ which are distorted to $|\tilde{\mu}_j\rangle$ as in (3). The effect of the operator $(1-\hat{K})^{-1}$ in (10) can be simulated by the normalization of the states $|\tilde{\mu}_j\rangle$ yielding $|\tilde{\mu}_{j,\mathrm{norm}}\rangle$. Therefore one expects the spectroscopic factor to behave roughly like

$$S \sim \prod_{j=1}^{a} |\langle \tilde{\mu}_{j,\mathrm{norm}} | \nu_{A+j} \rangle|^2. \tag{12}$$

This expression shows (*i*) that many single particle states contribute to S, and (*ii*) that S will roughly scale with

the a-th power of the square of a single particle overlap. The first point can be seen from the product (12) and from the structure (3) of the Pauli distortions.

Since many single particle states contribute to S one may expect that in general the resulting values reflect *bulk* properties of the involved nuclei. At the same time it means that uncertainties in the structure of the decaying nucleus may imply relatively large errors in S: For example, a change of the single particle overlaps by 10% yields a factor 30 in S for ^{16}O-decay. The uncertainties of the microscopically calculated values are of one to two orders of magnitude for the region $a \approx 16$. Like S itself, the uncertainties will also roughly scale with the power a. Explicit microscopic calculations are therefore of limited use for predicting heavier decay modes. What they do show, however, is that a description based on (1) yields the right magnitude for the absolute values.

Formula (12) indicates that, compared to α-spectroscopic factors S_α, the spectroscopic factor for heavier fragments should scale like $(S_\alpha)^{a/4}$. The comparison with our microscopic calculations shows [20] that the scaling is much better described by

$$S_{\text{bulk}} = (S_\alpha)^{(a-1)/3} \quad \text{(bulk spectroscopic factor).} \tag{13}$$

This can be easily understood by noticing that the centre-of-mass motion is (and should be) split off from the shell model function of the light fragment. Consequently the ϕ_a depends on $a-1$ internal coordinates only; each coordinate yields one factor in the product (12). This explains the scaling with $(a-1)/3$ instead of $a/4$.

The relatively large number of contributing single particle functions, the inherent uncertainties and the calculational effort of such a calculation suggest to use the *bulk spectroscopic factor* (13) for practical purposes. This will be done in the following. It should, however, be kept in mind that the justification of (6) with (13) is based on the microscopic starting point (1) and corresponding explicit microscopic calculations [6, 17, 18].

2.3. *One-body decay constant*

The one-body decay constant λ_G is determined by the Schrödinger equation (5). In this Schrödinger equation we use the semiempirical heavy ion potential

$$V(R) = -\frac{50\ \text{MeV}}{\text{fm}} \frac{R_a R_A}{R_a + R_A} \exp\left(\frac{R_a + R_A - R}{d}\right) + \frac{zZe^2}{R},$$
$$d = 0.63\ \text{fm},\ R_n = (1.233 n^{1/3} - 0.978 n^{-1/3})\ \text{fm},\ n = a \text{ or } A \tag{14}$$

which is fitted to elastic scattering data [19]. A centrifugal term might be added in (14). Such a term as well as the deviation of the Coulomb potential from zZe^2/R have only negligable effects on the following calculation; therefore they are omitted here. The potential (14) is unrealistic in the interior but it is used only outside the inner turning point. Because of its applicability to all kinds of light fragments this potential is well-suited for the intended unified description of α- and exotic decays.

Instead of solving the Schrödinger equation (5) exactly we calculate λ_G by the following semiclassical WKB approximation:

$$\lambda_G \simeq \frac{v}{2R_i} \exp\left(-2 \int_{R_i}^{R_0} \mathrm{d}R \sqrt{\frac{2\mu}{\hbar^2}[V(R) - E_a]}\right). \tag{15}$$

Here R_i and R_0 are the inner and outer turning points, and μ is the reduced mass. The tunneling energy E_a of the emitted cluster (the asymptotic energy of the relative motion) is given by

$$E_a = (M_{A+a} - M_A - M_a)\, c^2 \tag{16}$$

if all nuclei are in their ground states. The M_n are the rest masses of the bare nuclei stripped from electrons. Since usually masses of neutral atoms are listed [21] the extracted Q-values have to be corrected for the small electronic binding energies [22]. This correction increases the calculated penetrabilities by about a factor of 3.

For the prefactor ('knocking frequency') in (15) we use

$$v = \frac{v}{2R_i} = \sqrt{\frac{a \cdot 25\ \text{MeV}}{2\mu R_i^2}}. \tag{17}$$

This corresponds to a kinetic energy $\mu v^2/2 = a \cdot 25$ MeV of the emitted fragment inside the barrier, or to a typical value of 100 MeV for the potential depth of an α-nucleus potential. Since the reduced mass μ scales with a the prefactor (17) is nearly constant for all decay modes.

The uncertainty of the calculated decay constants λ_G is due to the specific choice of the potential $V(R)$. Comparing the results for various realistic potentials leads to an estimate of the corresponding error; this error is about a factor of 2 for λ_G. It includes the uncertainty of the kinetic energy inside the potential (related to the potential depth).

Sometimes it is claimed that the semiclassical treatment implies large errors. Since the procedure involves turning points an improper application of the WKB approximation may indeed yield arbitrary results. The expression (15) is, however, well-behaved and deviates from the exact solution of (5) by less than a factor of 2 (Appendix A). This error is comparable to that due to the uncertainties of $V(R)$, and it is small compared to the uncertainties of microscopically determined spectroscopic factors. An exact solution of (5) is therefore of no advantage.

3. Results

3.1. *Unified decay constant*

Equation (6) with (15) and (13) yields the final formula for the decay constant used in our unified model:

$$\lambda_{\text{unified}} = \lambda_G(E_a)\, S_{\text{bulk}}(a) = \lambda_G(E_a)\, S_\alpha^{(a-1)/3}$$
(unified decay constant). (18)

Here the sensitive dependences on E_a and on a are explicitly displayed but not the less sensitive ones on A and Z. The input of the unified decay constant is completely fixed by the following quantities:

- Mass and proton numbers of the fragments, A, Z, a and z.
- Asymptotic energy E_a of the relative motion of the fragments.
- Semiempirical potential $V(R)$ as defined in (14).
- One parameter S_α, for odd and even decay modes, respectively.

If we used the S_α of existing microscopic calculations there would be no adjustable parameter in the expression for λ_{unified}. For an optimal reproduction of all data it is, however, preferable to treat S_α as a fit parameter. This parameter S_α is determined by the fit to decay constants up to $a=28$:

$$S_\alpha = \begin{cases} S_\alpha^{\text{even}} = 6.3 \cdot 10^{-3} & \text{even nuclei} \\ S_\alpha^{\text{odd}} = 3.2 \cdot 10^{-3} & \text{odd nuclei} \end{cases} \quad \text{(fitted).} \tag{19}$$

The fitted values are in agreement with theoretical and experimental α-spectroscopic factors in the Pb region. Because the expression (18) for the decay constant can be easily evaluated we do not present an exhaustive list of decay constants. On request the computer code realizing (18) is available for an IBM compatible PC on a $5\frac{1}{4}''$ discette (360 kB or 1.2 MB).

3.2. Test of the model

Our model is extinguished from other ones by depending on 1 parameter (for even and odd decay modes, respectively) only. This parameter cannot be considerably altered by new experimental data.

First results of the unified formula (18) have been presented in a previous letter [20]. It has been shown in [20] that all decay constants known at that time are well reproduced (within a factor of 4) and that our results are compatible with all experimental limits.

However, the crucial test of such a model consists in predicting numbers for unmeasured decays. Since the publication of the first results [20] some new decay modes have been measured. The predictions of our model are compared with the experimental values in Table 1.

Table 1. Predicted half-lives $\tau(a)$ from [20]. These values were subsequently confirmed by experiment within the range ± 0.5 for $\log \tau$

$^{A+a}(Z+z) \rightarrow {}^{a}z$	$+{}^{A}Z$	E_a [MeV]	$\log \tau(a)$	$\log \tau_{\text{exp}}(a)$	[Ref.]
^{236}Pu	$\rightarrow {}^{28}$Mg $+{}^{208}$Pb	79.90	21.3	21.7	[12]
^{238}Pu	$\rightarrow {}^{32}$Si $+{}^{206}$Hg	91.47	25.8	25.3	[9]
	$\rightarrow {}^{28}$Mg $+{}^{210}$Pb	76.16	25.6		
				25.7	[9]
	$\rightarrow {}^{30}$Mg $+{}^{208}$Pb	77.26	25.8		

The agreement (within a factor of 4) constitutes a successful test of the model.

3.3. Model predictions

For some yet unmeasured decay modes the numerical values of the predicted half-lives are listed in Table 2. The branching ratios $B=\tau(\alpha)/\tau(a)$ relative to α-decay are also listed. From Table 1 and previous results we attribute a typical error of about a factor of 4 to these predictions.

In Table 2 we selected mainly decays for which a detection appears possible. The present borders of detectability are $\log \tau \leqq 29$ and $B \gtrsim 10^{-16}$; an improvement to $B \gtrsim 10^{-18}$ seems to be conceivable [9]. Especially the decay of ^{229}Th is interesting because besides the α-decay three different decay modes should be detectable, namely the emission of ^{14}C, ^{20}O and ^{24}Ne nuclei.

According to our model the observation of decay modes with emitted fragments heavier than ^{34}Si seems to be hopeless. However, with increasing a our model is expected to become eventually invalid. Therefore the last two entries of Table 2 for ^{46}Ar do only mean that a mechanism comparable to α-decay does not lead to an observable decay. For sufficiently heavy fragments fission as a different mechanism will, however, set in.

Table 2. Predictions of (18) for the half-lives $\tau(a)$ of some yet unmeasured decay modes. Here $\tau(\alpha)$ is the experimental half life of the competing α-decay and $B=\tau(\alpha)/\tau(a)$ the corresponding branching ratio. The typical uncertainty of $\tau(a)$ is a factor of 4. The last two entries should be considered with caution since our model might be at the border of validity for this heavy fragment

$^{A+a}(Z+z) \rightarrow {}^{a}z$	$+{}^{A}Z$	E_a [MeV]	$\log \tau(a)$	$\log \tau(\alpha)$	$-\log B$
^{228}Th	$\rightarrow {}^{14}$C $+{}^{214}$Po	28.34	23.0	7.78	15.2
^{229}Th	$\rightarrow {}^{20}$O $+{}^{209}$Pb	43.57	25.9	11.36	14.5
	$\rightarrow {}^{14}$C $+{}^{215}$Po	27.22	26.9		15.5
	$\rightarrow {}^{24}$Ne $+{}^{205}$Hg	58.02	27.0		15.6
^{230}U	$\rightarrow {}^{22}$Ne $+{}^{208}$Pb	61.59	20.4	6.25	14.1
	$\rightarrow {}^{24}$Ne $+{}^{206}$Pb	61.55	22.2		16.0
	$\rightarrow {}^{14}$C $+{}^{216}$Rn	28.47	24.7		18.5
^{231}Pa	$\rightarrow {}^{23}$F $+{}^{208}$Pb	52.01	24.7	12.0	12.7
^{232}U	$\rightarrow {}^{28}$Mg $+{}^{204}$Hg	74.54	25.3	9.36	15.9
^{235}U	$\rightarrow {}^{24}$Ne $+{}^{211}$Pb	57.55	29.9	16.33	13.6
	$\rightarrow {}^{28}$Mg $+{}^{207}$Hg	72.42	30.5		14.2
	$\rightarrow {}^{26}$Ne $+{}^{209}$Pb	58.30	30.9		14.6
^{236}U	$\rightarrow {}^{30}$Mg $+{}^{206}$Hg	72.73	29.0	14.9	14.1
	$\rightarrow {}^{24}$Ne $+{}^{212}$Pb	56.15	29.8		14.9
	$\rightarrow {}^{26}$Ne $+{}^{210}$Pb	56.94	30.5		15.6
^{239}Pu	$\rightarrow {}^{34}$Si $+{}^{205}$Hg	91.11	30.9	11.88	19.0
^{240}Pu	$\rightarrow {}^{34}$Si $+{}^{206}$Hg	91.31	27.3	11.31	16.0
^{240}Cm	$\rightarrow {}^{32}$Si $+{}^{208}$Pb	97.83	21.8	6.37	15.4
^{242}Cm	$\rightarrow {}^{34}$Si $+{}^{208}$Pb	96.81	24.1	7.15	16.9
^{243}Cm	$\rightarrow {}^{34}$Si $+{}^{209}$Pb	95.05	29.1	8.95	20.2
^{251}Cf	$\rightarrow {}^{46}$Ar $+{}^{205}$Hg	126.51	34.9	10.45	24.4
^{252}Cf	$\rightarrow {}^{46}$Ar $+{}^{206}$Hg	127.07	29.9	7.91	22.0

3.4. Fine structure

Following Hussonnois et al. [25] and Mang [5] we present a qualitative discussion of the fine structure [11] of the ^{14}C-decay of ^{223}Ra. Most decays (81%) lead to the first excited state of ^{209}Pb at 0.779 MeV although the penetrability for the decay to the ground state is 32 times larger. Clearly, this phenomenon must be connected to structure effects, that means in our model to the spectroscopic factor.

Our microscopic calculations (using the spherical shell model) as well as the expression for λ_G (using a spherical potential $V(R)$) refer to spherical nuclei. In spite of this simplification the gross features of exotic decays could be reproduced over a wide range of nuclei. For the explanation of the fine structure the deformation has to be taken into account. On the microscopic level this can be done by using Nilsson shell model states which are classified by the usual quantum numbers $K^{\pi}[Nn_z\Lambda]$. The transitions between states with identical Nilsson quantum numbers are favoured because these states have enhanced overlaps leading to larger spectroscopic factors. As stated by Mang [5] for α-decay, the favoured *odd* decays behave very much like *even* transitions.

The parent nucleus ^{223}Ra may be described in the rotor model by the mixed parity doublet $3/2^{\pm}[631]\otimes[761]$. The ground state of the daughter nucleus ^{209}Pb has the quantum numbers of the odd neutron in $2g9/2$ (single particle level of the spherical shell model), the first excited state at 0.779 MeV that of $1i11/2$ and the second excited state at 1.422 MeV that of $1j15/2$. In contrast to the ground state, the first two excited states contain large components $3/2^{+}[631]$ and $3/2^{-}[761]$, respectively [25]. Therefore, the transition into these excited states is favoured by the internal structure. The actual branching ratio of the two excited states (roughly 40 to 1) is caused by the different penetrabilities.

If favoured odd decays behave similar as even decays we should use S_{α}^{even} rather than S_{α}^{odd} in (18). With this specification and using the appropriate penetrabilities (excitation energies substracted from E_a), the favoured ^{14}C-decays of ^{223}Ra are indeed quantitatively reproduced by λ_{unified}.

This discussion leads to the following generalizations and conclusions:

1. As for α-decay, it can be expected that most exotic decays from an odd nucleus will end up in excited states of the (odd) daughter nucleus. As long as no information about the fine structure is available (from experiment or from arguments like the ones presented above) we have to use the energy E_a of (16) in (15). This energy is too large because the (unknown) excitation energy is not substracted. This means that for the reproduction of the favoured odd decays a too large penetrability is inserted in (18). This is effectively compensated for by the smaller value of the fitted S_{α}^{odd} (as compared to S_{α}^{even}).

2. If, at a later time, the fine structure is measured for many exotic decays the unified decay constant should be used in a somewhat modified way: With the parameter S_{α}^{even} renamed as $S_{\alpha}^{\text{favoured}}$ and with the correct penetrabilities (excitation energies substracted from E_a), the expression (18) with $S_{\alpha}^{\text{favoured}}$ applies then to even *and* to favoured odd decays simultaneously.

3. Besides the even and the favoured odd decays (point 2) there are *unfavoured odd* decays. Their spectroscopic factors can probably not be reproduced by a bulk formula like (13) because they might depend sensitively on the specific structure of the parent and daughter nucleus. Even if this structure were known microscopic calculations of the spectroscopic factors would be very cumbersome. In particular the quite different deformations of the parent and daughter nucleus complicate such calculations.

We conclude this subsection by stating that a qualitative understanding of the fine structure is possible in the framework of our model. No such explanation can be given within macroscopic models (Sect. 4).

4. Other approaches

The generalization of the microscopic α-decay theory leads to a successfull explanation of exotic decays. This does not exclude the possibility of other descriptions. The obvious alternative is the treatment of exotic decays as an extremely asymmetric cold fission. A realistic microscopic approach to fission would require a multi-dimensional parameter space. However, all existing fission approaches to exotic decays are restricted to one degree of freedom; they are thus macroscopic one-body models.

In these models, the deformation energy of the fissioning nucleus is parametrized versus the distance between the fragment centers or some related quantity. For separated fragments the deformation energy becomes the Coulomb plus centrifugal potential. Apart from the Q-value no information about the microscopic (nucleonic) structure of the nuclei enter into these models. The penetration of the classically forbidden deformation barrier determines the fission probability. The tunneling, usually described by the semiclassical WKB approximation, leads to a decay constant formula of the same structure as λ_G in (15).

Compared to our expression $\lambda=\lambda_G S=\nu PS$ all macroscopic one-body models [23] can be characterized by setting the spectroscopic factor $S=1$; the differences in the frequency ν play a minor role. To compensate for the missing factor $S\ll 1$ the potential barrier of these one-body models has to be larger than the one we used (see e.g. Fig. 1 in [9]). Since only the outer tail of the potential is well determined by scattering data, the interior part may be parametrized in many ways leading to the variety of one-body models. An unambiguous determination of these parametrisations from decay data seems to be impossible. Therefore, these parameters are usually fitted and updated in order to obtain agreement with known decays.

Among the various macroscopic models we discuss specifically the tuned Gamov factor model of Price [9] because it depends on 2 parameters only. Price's model

reproduces well the decay constants of exotic decays. It uses a Coulomb potential for $R>R_i$ and a square well potential for $R<R_i$ where $R_i(r_0)=r_0(a^{1/3}+A^{1/3})$. The decay constant is then calculated by

$$\lambda_{\text{Price}}=\nu \ln(2)\exp(-2I(r_0)) \tag{20}$$

where $\exp(-2I(r_0))$ is the penetration factor. In this model the values $r_0=0.928$ fm, $\nu_{\text{even}}=4.3\cdot10^{26}\ \text{s}^{-1}$ and $\nu_{\text{odd}}=1.1\cdot10^{25}\ \text{s}^{-1}$ for even and odd decay modes, respectively, are fitted to experiment. The frequencies differ by a factor of 10^4 from typical nuclear frequencies, and the radii are much smaller than measured nuclear sizes. For the expression (20) one can see how the spectroscopic factor is effectively simulated. If the semiempirical heavy ion potential (14) is approximated by a simple square well potential with a *realistic* radius $R_i(r_0)$ (see Appendix B) our unified decay constant (18) reads

$$\lambda_{\text{unified}}=\frac{\nu}{2R_i}S(a)\exp(-2I(r_0=1.286\text{ fm})). \tag{21}$$

By comparison with λ_{Price} we see that $S(a)$ is replaced by the factor

$$S_{\text{eff}}=\text{const.}\exp\left(-2\int_{R_i(r_0=0.928\text{ fm})}^{R_i(r_0=1.286\text{ fm})}\mathrm{d}R\cdot\sqrt{\frac{2\mu}{\hbar^2}\left(\frac{zZe^2}{R}-E_a\right)}\right) \tag{22}$$

which might be considered as an 'effective spectroscopic factor'. The additional area between the artificial $R_i(r_0=0.928$ fm) and the realistic $R_i(r_0=1.286$ fm) appearing in the penetrability compensates for the missing preformation probability.

From our point of view, all macroscopic models simulate the structure effects (the preformation probability) by an enhanced barrier. This kind of approach was also quite common in the early days of the α-decay theory.

5. Summary

In Sect. 1 it has been argued that the microscopic α-decay theory might be extended to exotic decays. The consequently obtained excellent agreement with the experimental data strongly supports the underlying physical assumption: The decay constant $\lambda=\lambda_G S$ is given by the one-body decay constant λ_G times the preformation probability S. From α- to Si-decay this preformation probability varies over 21 orders of magnitude from 10^{-2} to 10^{-23}. The order of magnitude of this factor and its variation can be understood microscopically.

Our simple unified model covers a range of exotic decays with emitted fragment mass numbers from $a=4$ to $a=34$, with half-lives from 10^{11} s to 10^{28} s, and with branching ratios relative to α decay from 10^{-9} to 10^{-16}. In addition, it provides a qualitative understanding of the fine structure effects.

Appendix A: Errors of the WKB approximation

We want to compare the semiclassical WKB-approximation (15) with the exact solution of the Schrödinger equation (5). For this purpose the nuclear (V_N) and the Coulomb (V_C) part of the potential

$$V(R)=V_N(R)+V_C(R) \tag{A.1}$$

has to be defined in a sensible way also in the interior; the original potential (14) is good for the barrier region only. For the nuclear part we use

$$V_N(R)=-\frac{50\text{ MeV}}{\text{fm}}\frac{R_aR_A}{R_a+R_A}\frac{1}{c/a+\exp[(R-R_a-R_A)/d]}, \tag{A.2}$$

The radii R_a, R_A and the diffuseness d are the same as in (14). For $c=0$ the potential (A.2) reduces to the nuclear part of (14). The choice $c=2R_aR_A/(R_a+R_A)/\text{fm}$ meets two requirements: Firstly, the potential (A.1) approximates (14) well for $R\gtrsim R_a+R_A$. Secondly, the potential V_N has a finite depth which leads to the prefactor (17) (using $\mu v^2/2\cong -V_N(0)$).

The Coulomb part of (14) is replaced by that of a pointlike charge (ze) inside a homogeneously charged sphere (Ze) with $R_c=R_a+R_A$.

$$V_C(R)=zZe^2\begin{cases}\frac{3}{2R_c}\left(1-\frac{R^2}{3R_c^2}\right) & (R<R_c)\\ \frac{1}{R} & (R>R_c).\end{cases} \tag{A.3}$$

The potential (A.1) with (A.2) and (A.3) is well-defined in the interior so that both, the WKB approximation as well as the exact solution of (5), can be determined. The exact λ_G can be written [24] in the following form,

$$\lambda_G=\frac{2E_a}{\hbar k}\frac{|u(R_0)|^2}{G(\eta,kR_0)^2},\quad k=\sqrt{\frac{2\mu E_a}{\hbar^2}},\quad \eta=\frac{zZe^2}{2E_a}. \tag{A.4}$$

For this expression the numerically determined real 'bound state' solution $u(R)=R\varphi_{E_a}$ has to be normalized

Table 3. The decay constants λ_G from the exact solution of the Schrödinger equation are compared to λ_{WKB} given by the WKB approximation. For typical energies of exotic decays they differ by less than a factor of 2

Decay mode	E_a (MeV)	λ_G (s^{-1})	λ_{WKB} (s^{-1})	$\lambda_{\text{WKB}}/\lambda_G$
$^{222}\text{Ra}(^{14}\text{C})$	32.58209	$1.7\cdot10^{-4}$	$2.3\cdot10^{-4}$	1.4
$^{232}\text{U}(^{24}\text{Ne})$	60.43211	$4.4\cdot10^{-8}$	$6.8\cdot10^{-8}$	1.5
$^{236}\text{Pu}(^{30}\text{Mg})$	71.45492	$6.1\cdot10^{-11}$	$1.1\cdot10^{-10}$	1.8

to $\int_0^{R_0} dR|u(R)|^2 = 1$. The irregular Coulomb function is denoted by G. The radius R_0 must be chosen outside the range of the nuclear interaction but still inside the Coulomb barrier. The result does then not depend on the specific value of R_0. For the very sharp resonances considered here this formula is practically exact; its error is of order $\hbar\lambda/E$.

For some typical cases, Table 3 compares the exact results λ_G with the WKB approximation called λ_{WKB} here. The deviations are within the errors due to the uncertainties of the potential (Subsect. 2.3). The use of the WKB approximation (15) is therefore adequate for the considered decays.

Appendix B: Analytical decay constant formula

In this Appendix we provide a simple analytical formula for the decay constant. This hand-pocket formula may be used for α-decay and all exotic decays. For this purpose the realistic potential (14) is simulated by a suitable square well potential with the radius

$$R_i = r_0(a^{1/3} + A^{1/3}), \qquad r_0 = 1.286 \text{ fm}. \tag{B.1}$$

The nearly a-independent prefactor $v/(2R_i)$ is fixed to $1.9 \cdot 10^{21}\ s^{-1}$. The resulting decay constant λ_{sq} is then given by the (hand-pocket formula):

$$\lambda_{\text{unified}} \simeq 1.9 \cdot 10^{21}\ s^{-1}\, S_\alpha^{(a-1)/3} \exp(-2I) \tag{B.2}$$

$$I = 0.315\, zZ \sqrt{\frac{aA}{a+A}} \sqrt{\frac{\text{MeV}}{E_a}} (\arccos(\sqrt{x}) - \sqrt{x - x^2})$$

$$x = 0.893 \frac{a^{1/3} + A^{1/3}}{zZ} \frac{E_a}{\text{MeV}}$$

where an effective nucleon mass of 931.1 MeV is used.

The input for this formula is A, Z, a, z, E_a of (16) and S_α of (19). All measured exotic decay constants are reproduced by formula (B.2) within a factor of 6.

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