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MICROSCOPIC CALCULATION OF THE ^{14}C DECAY OF Ra NUCLEI

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Abstract: Using microscopic wave functions the probability for finding the ^{14}C-daughter nucleus structure in Ra nuclei is calculated. Together with the penetrability computed for a phenomenological ^{14}C-daughter nucleus potential this yields a theoretical estimate of the decay width.

1. Introduction

In 1984 Rose and Jones [1]) found a new form of radioactivity: The decay of ^{223}Ra nuclei by the emission of ^{14}C fragments. Since then a number of such decays have been reported [2–4]). There are two obvious models for the theoretical description of such a process: It might be treated as a case of asymmetric fission or, alternatively, as a generalization of α-decay. First quantitative calculations using macroscopic models of this kind have led to the prediction of such decay modes by Sandulescu, Poenaru and Greiner [5]).

For a fission theory one needs a potential landscape of the parent nucleus in which various paths lead to the ^{14}C- and to the competing α-decay. It can be expected that the decay widths Γ will depend sensitively on the potential and mass parameters describing the dynamics in this space of collective variables. This provides a major difficulty for such an approach because no direct experimental information about these parameters is available (except for the asymptotic region).

In a treatment analogous to an α-decay process only the (experimentally reasonably well determined) surface region of the potential in the decay channel is important (for calculating the penetrability). In a microscopic model the interior region is then described by many-body wave functions of the nuclei considered. This picture leads to the concept of the preformation probabilities of the α- or ^{14}C-nucleus in the parent nucleus.

It is the aim of this paper to calculate these preformation probabilities on the basis of microscopic wave functions. For this purpose we use shell model functions for ^{14}C and the lead nuclei, and allow for pairing correlations in the wave functions

for the Ra nuclei. Using these wave functions we calculate the widths for the following decay processes:

$$^{222,223,224}\mathrm{Ra} \rightarrow \begin{cases} ^{14}\mathrm{C} + ^{208,209,210}\mathrm{Pb} \\ \alpha + ^{218,219,220}\mathrm{Rn}\,. \end{cases} \tag{1.1}$$

2. Model

2.1. OUTLINE OF THE MODEL

For our model of the ^{14}C decay we consider the following microscopic wave functions: ϕ_{A+14} for the ground state of the parent nucleus and $\phi_E = \mathcal{A} u_E(\boldsymbol{r})\phi_{14}\phi_A$ for the open ^{14}C-daughter nucleus channel. Here ϕ_{14} and ϕ_A are the internal microscopic ground state wave functions of ^{14}C and of the daughter nucleus, respectively. The difference between the centre-of-mass (c.m.) coordinates of ^{14}C and the daughter nucleus is denoted by $\boldsymbol{r}$ and the antisymmetrizer by $\mathcal{A}$. The function $u_E(\boldsymbol{r})$ describing the relative motion in this open channel will not be needed explicitly because this part of the process will be treated macroscopically. The open channel state can be written as $|\phi_E\rangle = \int \mathrm{d}\boldsymbol{R}\, u_E(\boldsymbol{R})|\boldsymbol{R}\rangle$ where we introduced a parameter $\boldsymbol{R}$ and states $|\boldsymbol{R}\rangle$ defined by

$$\langle \boldsymbol{r}_i | \boldsymbol{R}\rangle = \mathcal{A}\delta(\boldsymbol{r} - \boldsymbol{R})\phi_{14}\phi_A\,. \tag{2.1}$$

This is the coordinate representation of the states $|\boldsymbol{R}\rangle$ (the nucleon coordinates are denoted by $\boldsymbol{r}_i$). The set $\{|\boldsymbol{R}\rangle\}$ of states (2.1) (for different $\boldsymbol{R}$'s) spans the subspace of antisymmetrized ^{14}C-daughter nucleus states.

The structure of the open channel for ^{14}C decay is defined by (2.1). We may now calculate the probability S that the parent nucleus is of this structure or, equivalently, the percentage of the state $|\phi_{A+14}\rangle$ which lies in the subsapce $\{|\boldsymbol{R}\rangle\}$. This preformation probability S is quantum mechanically defined by the expectation value

$$S = \langle \phi_{A+14} | \hat{P} | \phi_{A+14}\rangle \tag{2.2}$$

of the projection operator $\hat{P}$ onto the subspace of states $|\boldsymbol{R}\rangle$. The operator $\hat{P}$ is given by

$$\hat{P} = \int \mathrm{d}\boldsymbol{R} \int \mathrm{d}\boldsymbol{R}'\, |\boldsymbol{R}\rangle (\hat{N}^{-1})_{\boldsymbol{R},\boldsymbol{R}'} \langle \boldsymbol{R}'|\,. \tag{2.3}$$

The Pauli principle together with the non-orthogonality of the nucleon states in ϕ_{14} and ϕ_A implies that the states $|\boldsymbol{R}\rangle$ are not orthonormal. Their overlap defines the norm kernel $\hat{N}$ (with $\hat{N} \neq 1$):

$$N(\boldsymbol{R}, \boldsymbol{R}') = \hat{N}_{\boldsymbol{R},\boldsymbol{R}'} = \langle \boldsymbol{R} | \boldsymbol{R}'\rangle\,. \tag{2.4}$$

The inverse norm kernel $\hat{N}^{-1}$ in (2.3) is necessary in order to ensure that $\hat{P}$ has the required property of a projector ($\hat{P}^2 = \hat{P}$).

The quantity S is also called spectroscopic factor. (For a possible deviation in nomenclature see sect. 5). According to eqs. (2.1)-(2.4) this S is well-defined once the microscopic wave functions ϕ_{14}, ϕ_A and ϕ_{A+14} have been specified.

We have now written down the mathematical expression for the probability of finding the structure of the open channel preformed in the parent nucleus. In order to connect this probability S with the decay constant we need the following two physical assumptions (which underly in this or a related form any existing microscopic calculation of α-decay processes):

(i) Only that part of ϕ_{A+14} which is of the structure (2.1) contributes to the ^{14}C decay.

(ii) For this part of ϕ_{A+14} the relative motion of ^{14}C and the daughter nucleus can be described by a phenomenological potential $U(R)$ fitted to elastic scattering. The second assumption means that the motion in the elastic channel ϕ_E itself is treated macroscopically. Due to this second assumption the decay constant $\lambda = \Gamma/\hbar$ is proportional to the λ_0 calculated from the one-body Schrödinger equation with $U(R)$. Due to the first assumption the proportionality factor is given by the S of (2.2). Therefore

$$\lambda = \lambda_0 S . \tag{2.5}$$

For the competing α-decay of the parent nucleus ϕ_{A+14} the above formulae apply and will be used accordingly: In (2.1) $\phi_{14}\phi_A$ has to be replaced by $\phi_4\phi_{A+10}$, and $\boldsymbol{r}$ denotes the corresponding difference of c.m. coordinates.

By the simple ansatz (2.5) we circumvent some difficulties encountered in more sophisticated reaction theories. According to our experience in α-decay calculations [6]) the use of more elaborate but still feasible expressions for λ does not lead to a reduction of the uncertainties (see sect. 3) contained in such a kind of theoretical estimate.

2.2. PREFORMATION PROBABILITY S

In this section we specify the microscopic many-body wave functions necessary for the calculation of S.

We use the single particle states $|\nu\rangle = |nljm\rangle$ of a harmonic oscillator shell model (SM). The oscillator constants β for the light fragments (ϕ_4 or ϕ_{14}) are adjusted to the corresponding experimental root-mean-square radii ($\beta_4 = 0.514\,\mathrm{fm}^{-2}$ and $\beta_{14} = 0.350\,\mathrm{fm}^{-2}$), for the heavy nuclei (ϕ_{A+14}, ϕ_{A+10}, ϕ_A) we use $\beta = 0.168\,\mathrm{fm}^{-2}$. The light fragments are described by the following SM configurations

$$|\alpha\rangle = |(1s)^2_p(1s)^2_n\rangle ,$$

$$|^{14}C\rangle = |((1s)^2(1p_{3/2})^4)_p((1s)^2(1p_{3/2})^4(1p_{1/2})^2)_n\rangle . \tag{2.6}$$

The indices p and n refer to the proton and neutron configurations, respectively.

All heavy nuclei contain the closed shell lead core $|^{208}\mathrm{Pb}\rangle$. The state for ^{209}Pb is then

$$|^{209}\mathrm{Pb}\rangle = a^+_{jm}|^{208}\mathrm{Pb}\rangle\,, \tag{2.7}$$

where a^+_{jm} is the single particle creation operator for the $|\nu\rangle = |2g_{9/2}\rangle$ neutron state.

In order to construct the other states of the heavy nuclei we introduce pair creation operators $A^+(j)$ creating two protons (or neutrons) of a specific j-shell in a $J = 0$ state:

$$A^+(j) = \sum_{m>0} (-)^{j-m} a^+_{jm} a^+_{j-m}\,. \tag{2.8}$$

In the pairing model several j-shells contribute to the actual pair states (the shells in the range $\varepsilon_F \pm \Delta$ where ε_F is the Fermi energy and Δ the gap energy). Treating the contributing j-shells as degenerate the normalized creation operator A^+ of such pair states reads

$$A^+ = \Omega^{-1/2} \sum_k A^+(j_k)\,, \qquad \Omega = \sum_k \tfrac{1}{2}(2j_k + 1)\,. \tag{2.9}$$

Here the sum over k runs over all contributing j-shells. For a simple ansatz of the many-body state we restrict ourselves to the following shells

$$\{j_k\} = \begin{cases} (1h_{9/2}),\ (2f_{7/2}) & \text{for protons} \\ (2g_{9/2}),\ (3d_{5/2}),\ (1i_{11/2}) & \text{for neutrons}\,. \end{cases} \tag{2.10}$$

The pairing model describes the ground state structure of the nucleons outside the closed shell by a product of the appropriate number of pair creation operators A^+_p and A^+_n (for protons and neutrons according to (2.10)). This is the seniority zero state for the even-even nuclei ^{222,224}Ra. The ground state of the isotope ^{223}Ra contains in addition one single-particle creation operator a^+_{jm} which for simplicity is taken to be the same as that in (2.7). The ground states of the parent nuclei are thus described by:

$$|^{223}\mathrm{Ra}\rangle = \mathrm{const}\,(A^+_n)^4 (A^+_p)^3 a^+_{jm}|^{208}\mathrm{Pb}\rangle\,,$$

$$|^{222(224)}\mathrm{Ra}\rangle = \mathrm{const}\,(A^+_n)^{4(5)} (A^+_p)^3|^{208}\mathrm{Pb}\rangle\,. \tag{2.11}$$

Because of the Pauli principle the A^+ violate (in general) the Bose commutation relations; this implies a non-trivial (however analytically calculable) normalization constant in (2.11). The considered ^{14}C decays lead to the daughter nuclei described by $|^{208}\mathrm{Pb}\rangle$, $a^+_{jm}|^{208}\mathrm{Pb}\rangle$ and $A^+_n|^{208}\mathrm{Pb}\rangle$; the competing α-decay leads to states of the form (2.11) with one A^+_n and one A^+_p less.

With the definition of all many-body states ((2.6, (2.7) and (2.11)) the spectroscopic factor S is fixed according to eqs. (2.1)-(2.4). The appendix summarizes a number of technical details of the actual computation of S.

2.3. DECAY CONSTANT λ_0

In this section we specify the input necessary for the calculation of λ_0.

The penetration of the performed ^{14}C (or α) through the Coulomb barrier is described in a Gamov picture, that means by a one-body Schrödinger equation with a phenomenological potential $U(R)$ fitted to elastic scattering. $U(R)$ is of the form

$$U(R) = V(R) + V_{\text{Coul}}(R) + V_L(R) . \tag{2.12}$$

The Coulomb potential V_{Coul} is taken to be that of a homogeneously charged sphere of proper size (asymptotically $Z_1 Z_2 e^2/R$). The centrifugal potential is given by $V_L = \hbar^2 L(L+1)/2M$ where L is the angular momentum of the relative motion and M the reduced mass. For the nuclear part we employ the following semi-empirical potential [7])

$$V(R) = -(50\ \text{MeV/fm})(R_1 R_2/(R_1+R_2))\exp(-(R-R_1-R_2)/a) ,$$

$$R_j = (1.233A_j^{1/3} - 0.978A_j^{-1/3})\ \text{fm} \qquad (j=1,2) , \qquad a = 0.63\ \text{fm} . \tag{2.13}$$

For the present purpose this potential has the advantage that it may be used for the α-channel as well as for the ^{14}C channel. For the α-case it yields penetrabilities which compare well with those of Woods–Saxon potentials found in the literature [8]). This potential is also tested for larger fragments and thus suitable for the ^{14}C channel. For the ^{14}C case we tried alternatively a Woods–Saxon potential with parameters ($V_0 = 210.7$ MeV, $r_0 = 1.1$ fm, $a = 0.6375$ fm) adjusted to ^{16}O–^{208}Pb scattering [9]). Compared to (2.13) it yields a deviation of a factor 2.5 to 3 in the decay constant which is within the range of uncertainty to be discussed more systematically in sect. 3.3.

The decay constant λ_0 is calculated in WKB approximation:

$$\lambda_0 = (v/2R_i)P, \qquad P = \exp\left(-2\int_{R_i}^{R_a} \mathrm{d}R\,[(2M/\hbar^2)(U(R)-Q_\alpha)]\right)^{1/2} . \tag{2.14}$$

Here R_i and R_a are the inner and outer turning points where $U(R) = Q_\alpha$. The asymptotic kinetic energy of the relative motion Q_α is determined by the known binding energies of the considered nuclei. For the assumed states (sect. 2.2) the angular momentum of the relative motion is $L = 0$. For the prefactor $v/2R_i$ ("knocking frequency") we assumed a kinetic energy $\frac{1}{2}Mv^2$ inside the barrier of 10^2 MeV for α and the scaled value $10^2(\frac{14}{4})$ MeV for ^{14}C (see also discussion in sect. 3.3).

3. Validity of the model assumptions

In sect. 2 we presented the model assumptions leading to $\lambda = \lambda_0 S$, and the necessary assumptions about the input for the actual calculation of λ_0 and S. In this section

we discuss the validity of these assumptions and present estimates of the possible errors in the calculated S and λ_0.

3.1. BASIC MODEL ASSUMPTION

For the relation $\lambda = \lambda_0 S$ we introduced in sect. 2.1 two assumptions, (i) the proportionality to the preformation probability S and (ii) the description of the penetration through the barrier in a one-body picture.

Point (i) contains the central assumption about the decay mechanism: S is the probability that the structure of the open channel ($\phi_E = \mathcal{A} u_E \phi_{14} \phi_A$) is already preformed in the parent nucleus. This channel is assumed to be the elastic channel (ϕ_{14} and ϕ_A are the ground states). Therefore this decay mechanism does not allow for a dynamic formation of the fragments during the process. (Note, however, that the internal structures of ϕ_{14} and ϕ_A are distorted in ϕ_E due to the Pauli principle).

It will be very hard to do a microscopic theory without this simplifying assumption. The proper generalization of the present model would consist in the inclusion of a sufficiently large set of additional microscopic states (excited states $\phi_{A+14,i}$ and $\mathcal{A} u_{E,ij} \phi_{14,i} \phi_{A,j}$). In view of the non-feasibility of such a more general model the basis model assumption about the decay mechanism can be tested only indirectly by comparing its consequences (our results) with the experiment.

With respect to this central assumption (process proportional to the preformation probability of the structure of the open channel) we note that it underlies any theory using a reduced amplitude or a spectroscopic factor. Also a reaction theory with a transition matrix element of a model hamiltonian H between ϕ_{A+14} and ϕ_E belongs into this category: Since H contains at most two-particle operators the matrix element is essentially determined by the overlap ($\propto S^{1/2}$) of the last 14 nucleon states in the bra and ket.

We come now to the second assumption (ii) which greatly simplifies the actual treatment of the elastic channel. According to (ii) this channel is treated in a one-body picture using a potential $U(R)$ deduced from fits to the elastic scattering. The uncertainties in this assumption are reflected by those in $U(R)$ which will be discussed in sect. 3.3.

In the following two ways one could try to replace the macroscopic treatment of the open channel according to (ii) by a more microscopic calculation:

One could attempt to solve the resonating group equation for $u_E(\boldsymbol{R})$ using a microscopic model hamiltonian. In practice this would only shift the uncertainties of $U(R)$ to that of the effective microscopic interaction and of other ingredients of the calculation. For the evaluation of the sensitive quantity P we prefer to minimize the uncertainty by using experimental information (potentials fitted to elastic scattering). Note also that any model must use the experimental decay energy Q_α: Due to the sensitivity of P on Q_α a microscopic calculation of Q_α would yield essentially arbitrary results.

A second possibility towards a more microscopic treatment of the open channel is the attempt to restrict the macroscopic picture to the post-contact region and to use the reduced amplitude $\Omega(\boldsymbol{R})$,

$$\Omega(\boldsymbol{R}) = \langle \boldsymbol{R} | \hat{N}^{-1/2} | \phi_{A+14} \rangle \tag{3.1}$$

in the interior. Such a procedure would, however, require a parent state ϕ_{A+14} which yields a sensible amplitude $\Omega(\boldsymbol{R})$ at least up to radii $R \approx R_i$. The bound SM states ϕ_{A+14} (for which the microscopic calculation of $\Omega(\boldsymbol{R})$ and S is feasible) can be a valid description only as long as the ^{14}C fragment is inside the parent nucleus (approximately for $R \lesssim 7$ fm). Therefore the open channel wave function has to be continued at least down to these radii. It is then simpler to divide the treatment into the two steps leading to $\lambda = \lambda_0 S$: One calculates first the width $\Gamma = \hbar\lambda_0$ for the solution $\varphi(\boldsymbol{R})$ of a one-body Schrödinger equation with the potential $U(R)$. For this purpose one may solve the time-dependent Schrödinger equation with the initial condition that $\varphi(\boldsymbol{R}, t=0)$ is localized inside the Coulomb barrier, this means that the integral $I(t=0) = \int \mathrm{d}\boldsymbol{R}\, |\varphi(\boldsymbol{R}, t=0)|^2$ over the interior region is 1. The time-dependent solution $\varphi(\boldsymbol{R}, t)$ yields $I(t) = \exp(-\lambda_0 t)$ and thus λ_0. The second step takes then into account the microscopic structure. Due to the given microscopic structure (ϕ_{A+14}) the probability of having initially the two considered fragments (properly antisymmetrized) is not 1 but S where

$$S = \int \mathrm{d}\boldsymbol{R}\, |\Omega(\boldsymbol{R})|^2 . \tag{3.2}$$

This definition of S is identical to that given in (2.2), (2.3). The function $\Omega(\boldsymbol{R})$ is localized in the interior because it is calculated from the bound model state ϕ_{A+14}. Conceptually the $\Omega(\boldsymbol{R})$ of the microscopic picture corresponds to the initial wave function $\varphi(\boldsymbol{R}, t=0)$ of the macroscopic or one-body picture.

3.2. UNCERTAINTIES IN THE CALCULATED S

With the definition of the wave functions the spectroscopic factor S is fixed. For the actual evaluation of the expression for S we used some essentially technical approximations which are described in the appendix. These approximations have been checked in similar systems by comparison to (more) exact calculations. The total error due to these approximations is estimated to be about a factor 2 for S.

The decisive source of possible errors in S is the uncertainty about the true structure of the nuclei, in particular of the nuclei far beyond the closed shell. We discuss the following four points (which may not be completely independent):

(i) The use of he harmonic oscillator SM instead of a more realistic SM.

(ii) The configuration mixing of the last 14 (or 4) nucleon states in the parent nucleus.

(iii) Possible structure effects in the remaining core.

(iv) The ground state spins.

For (i): For α-Pb and simple SM configurations the use of single-particle functions of a Woods-Saxon SM instead of the harmonic oscillator increases S by about a factor 2 (table 1 of ref. [10])). Scaling this result to the ^{14}C decay (see eq. (4.2)) we might expect an enhancement of about a factor 10 in $S(^{14}C)$.

For (ii): This point is again most extensively tested for α-^{208}Pb: Going from the simplest SM configuration of the nucleons outside the lead core to simple configuration mixing (comparable to (2.10)) enhances S by about a factor 5 which is only slightly increased by further configuration mixing [11]). (Also see ref. [11]) for a discussion of very large enhancement factors sometimes quoted in the literature). In the present calculation the pairing correlations could be extended by including further shells in (2.10). This might, however, increase the clustering of the nucleons outside the closed shell above its actual value because of the simplicity of our ansatz (eq. (2.9)). Such an extension had to be accompanied by the introduction of realistic mixing coefficients in (2.9).

For (iii): The definition of the states in sect. 2.2 implies that the daughter nucleus is contained as a core in the parent nucleus. That means that the core overlap of these A or $(A+10)$ neucleon states (which is contained in $\langle \boldsymbol{R} | \phi_{A+14} \rangle$) comes out as 1. Any difference in the core structure between the parent and daughter nucleus may lead to a substantial reduction of S because the core overlap involves many nucleons. In particular we might think of a difference of the internal deformation (which in principle can be expressed by a suitable configuration mixing in the spherical SM basis). To some degree such a difference of the internal deformation is to be expected (a) for the α-decay of Ra because we are far away from the closed shell and (b) for the odd isotope because the unpaired nucleon may induce different deformations (see next point (iv)).

For (iv): We assumed in ^{223}Ra, ^{219}Rn and ^{209}Pb the same single particle state for the unpaired nucleon. This is done for simplicity and because of the lack of unequivocal information about the ground state spins of ^{223}Ra and ^{219}Rn. For the reported [12]) spin assignments $\frac{5}{2}$ for ^{223}Ra and $\frac{1}{2}$ for ^{219}Rn we had to insert different states for the last nucleon in the parent and daughter nucleus. This leads to the following three effects: First, the emitted fragment must carry away an angular momentum $L \neq 0$ causing a moderate decrease of the penetrability and a corresponding increase of $S_{\exp}$ (70% for α and 60% for ^{14}C). Second, there will be a moderate change in S due to the difference in the single-particle states entering in the overlap $\langle \boldsymbol{R} | \phi_{A+14} \rangle$. Third, the unpaired nucleon introduces a deviation from the spherical symmetry due to its angular momentum $l \neq 0$. Such a deviation will induce some (small) intrinsic deformation which is now different for the parent and daughter nucleus. According to point (iii) this may then lead to a substantial reduction of S. Unfortunately there is no easy way to take into account this third effect in our microscopic calculation.

From the discussion of the points (i) to (iv) an error of a factor 10 does not seem unlikely in the calculated S. The possible enhancement due to (i) and (ii) is expected

to be larger for $S(^{14}C)$ than for $S(\alpha)$. A reduction due to (iii) is likely to occur for $S(\alpha)$ and for the odd isotope (see (iv)).

It will not be easy to obtain a drastic reduction of the discussed uncertainties by an improvement of the model input. In particular the technical difficulties in the computation of S (see for example the determination of $\hat{N}$ in the appendix) require the use of not too complicated wave functions.

3.3. UNCERTAINTY IN THE CALCULATED λ_0

We discuss the possible errors in λ_0 due to the following two points:

(i) Use of the WKB approximation in the form (2.14).

(ii) Uncertainty in the potential $V(R)$.

The first point can be checked by comparing the approximation (2.14) with exact solutions of the one-body Schrödinger equation. Such a comparison has been performed [13]) for a number of α-decays and several Woods–Saxon potentials (but not for the present choice (2.13) which is impracticable in the interior). The deviations between (2.14) and the exact solutions were found to be limited by about 50%. Since the penetrabilities P for the ^{14}C are of the same order of magnitude we may expect similar deviations in this case. Such errors are about one order of magnitude smaller than those introduced by the second point (ii); therefore it is of no advantage to replace the simple expression (2.14) by an exact numerical solution.

In view of this comparison with fully quantum mechanical solutions the appearance of the prefactor $v/2R_i$ in (2.14) does not imply the assumption of a corresponding classical motion. The simplified form (2.14) used here is of advantage for the discussion of the influence of the uncertainty of $V(R)$ which is the decisive origin of errors in λ_0. We discuss now the possible errors in the calculated λ_0 due to the uncertainties of $V(R)$ in the interior and around $R = R_i$.

Since the elastic scattering is not sensitive to the potential in the interior (because of the strong absorption) the potential depth is experimentally undetermined. This is also the reason for the possibility of the functional form of (2.13) which in contrast to a Woods–Saxon potential does not yield a sensible depth in the interior. For the experimentally undetermined potential depth we have to use a theoretical estimate. Fortunately, λ_0 is not very sensitive to this quantity so that the error due to an even crude estimate is tolerable. The values actually used in (2.14) (10^2 MeV for α and $10^2(\frac{14}{4})$ MeV for ^{14}C) correspond roughly to a folding potential. We expect that these values are sensible up to a factor 4 (assuming the pessimistic factor 10 would not greatly change the final uncertainty estimate). This yields a possible error of a factor 2 up or down in λ_0.

As it is well-known, the penetrability P is rather sensitive to small changes in the potential because $V(R)$ enters into the exponent. However, only values $V(R)$ for $R > R_i$ count. At the radius R_i the nuclei are just touching each other and start to feel the nuclear attraction ($|V(R_i)|$ is about 60% of $V_{Coul}(R_i)$ for ^{14}C). A few fermi

outside R_i the potential $U(R)$ is purely Coulomb and no uncertainty occurs. For a simple estimate of the uncertainty in P we argue as follows: The quantity which is reasonably well fixed by scattering experiments is the sum $R_0 = R_1 + R_2$ of the radii of the two nuclei. The uncertainty ΔR of R_0 will be somewhere between 0.1 and 0.5 fm. A difference ΔR leads to a change in the penetrability ($P \to P'$) of the order

$$P'/P \sim \exp(2q\Delta R), \tag{3.3}$$

where q is a typical value of the wave vector under the top of the barrier. For ^{14}C–Pb the height of the Coulomb barrier is $B \simeq 70$ MeV, and $\hbar^2 q^2/2M = B - Q_\alpha$ yields $q \simeq 5\,\mathrm{fm}^{-1}$. Using $\Delta R = 0.25$ fm the ratio P'/P comes out as 12. For a potential corresponding to an average value R_0 we obtain thus an uncertainty of a factor 3.5 up or down. Combining this uncertainty with that of the prefactor we estimate that our calculation yields λ_0 values which are realistic within a factor 4.

The above arguments show that there is indeed a considerable uncertainty in the calculated λ_0. We note, however, two points: First, the uncertainty is largely due to the penetrability and seems to be unavoidable in any kind of reaction theory. Second, the uncertainty is not arbitrarily large.

For an illustration of the second point we remark that a change of $\Delta R \sim 2.3$ fm is required in order to eliminate a preformation factor $S \sim 10^{-10}$ in (2.5) (which is as we will see a realistic value for S). Compared to the resulting factor (10^{10}) in P the change ΔR is indeed relatively small (25% of R_i). In the early days of α-decay calculations [14]) it was common to reproduce the experimental data by such an adjustment of "effective" nuclear radii (that means using $\lambda = \lambda_0$ or $S = 1$ in (2.5)). From our present knowledge of nuclear sizes such an attempt seems to be unrealistic.

Some non-negligable change in λ_0 may result from the effect of deformations on the barrier. Landowne *et al.* [15]) estimated that this effect yields an enhancement factor of 16 for $\lambda_0(^{14}\mathrm{C})$ and 1.2 for $\lambda_0(\alpha)$.

4. Results and discussion

4.1. RESULTS

Tables 1 and 2 summarize the results of the α- and ^{14}C-decay calculations for the nuclei ^{222,223,224}Ra. For the comparison we introduced the "experimental"

TABLE 1

The α-decay energies Q_α and decay widths $\Gamma_\alpha = \hbar\lambda_{\mathrm{exp}}$ for three Ra isotopes, along with the experimental preformation factors S_{exp} which are compared with the calculated spectroscopic factors

	Q_α (MeV)	Γ_α (MeV)	P	S_{exp}	S	S_{exp}/S
^{222}Ra	6.68	$1.20 \cdot 10^{-23}$	$3.1 \cdot 10^{-22}$	$1.5 \cdot 10^{-2}$	$5.9 \cdot 10^{-2}$	0.25
^{223}Ra	5.98	$4.63 \cdot 10^{-28}$	$2.7 \cdot 10^{-25}$	$6.9 \cdot 10^{-4}$	$5.4 \cdot 10^{-2}$	0.013
^{224}Ra	5.79	$1.47 \cdot 10^{-27}$	$3.3 \cdot 10^{-26}$	$1.8 \cdot 10^{-2}$	$6.7 \cdot 10^{-2}$	0.27

TABLE 2

Same as table 1 for the ^{14}C decay

	Q_{14} (MeV)	Γ_{14} (MeV)	$\Gamma_{14}/\Gamma_\alpha$	P	S_{exp}	S	S_{exp}/S
^{222}Ra	33.05	$4.44 \cdot 10^{-33}$	$3.7 \cdot 10^{-10}$	$8.0 \cdot 10^{-25}$	$2.5 \cdot 10^{-9}$	$1.8 \cdot 10^{-11}$	140
^{223}Ra	31.84	$2.78 \cdot 10^{-37}$	$6.0 \cdot 10^{-10}$	$6.1 \cdot 10^{-27}$	$2.0 \cdot 10^{-11}$	$1.5 \cdot 10^{-11}$	1.3
^{224}Ra	30.53	$6.32 \cdot 10^{-38}$	$4.3 \cdot 10^{-11}$	$2.1 \cdot 10^{-29}$	$1.3 \cdot 10^{-9}$	$1.3 \cdot 10^{-10}$	10

The small ratio $\Gamma_{14}/\Gamma_\alpha$ of the decay widths (of the order 10^{-10}) is predominantly due to the corresponding ratio of the preformation probabilities.

spectroscopic factor S_{exp} given by the ratio of the measured decay constant λ_{exp} to λ_0,

$$S_{exp} = \lambda_{exp}/\lambda_0 \tag{4.1}$$

This is a common procedure in cluster transfer reactions where these factors might be obtained for example by relating the experimental to the DWBA cross section.

In the interpretation (4.1) the experimental data provide a rather direct information about the preformation probabilities. For the considered decay modes (α- and ^{14}C-decay) these probabilities vary from 10^{-2} to 10^{-10}. The uncertainty in λ_0 (see sect. 3.3) can only slightly modify this variation over many orders of magnitude.

4.2. DISCUSSION

Roughly speaking the spectroscopic factor is proportional to the squared product of the overlaps between the nucleon states in the emitted fragment and the last 4 or 14 states in ϕ_{A+14}. (The actual structure is more complicated due to $\hat{N}^{-1}$ in eq. (2.3).) Disregarding the specific structure of these nucleon states one would therefore expect that the values for S scale as follows

$$S(^{14}C) \sim [S(\alpha)]^{14/4}. \tag{4.2}$$

Using a value $S_{exp}(\alpha) \sim 10^{-2}$ typically found for even nuclei in the lead region this crude estimate yields $S(^{14}C) \sim 10^{-7}$. The fact that the actual values for $S(^{14}C)$ are smaller indicates that the correlations of the relevant nucleons in ϕ_{A+4} tend to be more α-like than ^{14}C-like.

The microscopically calculated S roughly reproduces the variation (10^{-2} to 10^{-10}) of the experimental preformation factor. The Tables show, however, also two discrepancies: (a) the α-decay probabilities are overestimated in contrast to an underestimate for ^{14}C and (b) the actually observed reduction for the odd isotope is not reproduced. In view of the discussion of the points (i) to (iv) of sect. 3.2 the explanation is probably as follows:

More realistic correlations of the nucleons forming the fragment will increase mainly $S(^{14}C)$ (recall the factor 10 for Woods-Saxon functions). Slight differences

in deformation between the parent and daughter nucleus can substantially reduce the core overlap (which is 1 for the present states). According to point (iii) and (iv) of sect. 3.2 such an effect is to be expected for the α-decay and, in addition, for both decay modes of the odd isotope.

For the valuation of the theoretical results one should consider the following point: $S(^{14}C)$ is related to the squared product of the overlaps of the last 14 nucleon states (those of ^{14}C with the last 14 states in Ra). A factor 100 in S then corresponds to a change of only 18% in each single-particle overlap. Similarily, S depends sensitively on differences in the core structure (point (iii) of sect. 3.2). In view of this sensitivity of S the presented rough reproduction of the data is satisfying. The overall agreement may also be considered as a verification of the basic model assumption, namely that the decay width is proportional to the degree to which the structure of the open channel is preformed in the parent nucleus.

This reproduction of the experimental data can, of course, not exclude the possibility of a different decay mechanism. In this connection we note that also the macroscopic fission model introduced by Shi and Swiatecki [16]) accounts for the data. In this model the formation of the fragments is preceded by the deformation of the parent nucleus. Since this deformation regime is energetically forbidden it contributes to the penetrability integral. The deformation part of the potential is smoothly connected to the surface region which is described by the proximity interaction. For this potential the penetrability P is calculated with the same expression (2.14) as used here. Due to the very different potential the resulting values for P are smaller than ours by roughly a factor of order 10^{10}. Since the physical assumptions of this and of our model are rather different we do not see how a connection between this factor 10^{-10} and our microscopically calculated preformation probabilities could be made.

5. Conventional spectroscopic factor

The meaning of the experimental spectroscopic factor $S_{\exp}$ is fixed by its relation (4.1) to the experimental decay constant. This concept of $S_{\exp}$ is used in an analogous way for α-transfer reactions. Such a procedure fixes the meaning of the spectroscopic factor to the degree to which the underlying one-body model (λ_0 in (4.1) or the DWBA cross section) is well-defined.

Sect. 2.1 explained (see eq. (2.5)) that the theoretical S of (2.2) is the quantity to be compared to $S_{\exp}$. In publications before 1975 (and partly also later) a different theoretical quantity was considered as the appropriate spectroscopic factor and compared to $S_{\exp}$. We call this different quantity "conventional spectroscopic factor" S_{conv}; comparisons between S and S_{conv} may be found in ref. [17]). In the notation of sect. 2.1 the conventional spectroscopic factor is given by

$$S_{\mathrm{conv}} = \langle \phi_{A+14} | \hat{O} | \phi_{A+14} \rangle \qquad \text{with } \hat{O} = \int \mathrm{d}\boldsymbol{R}\, |\boldsymbol{R}\rangle\langle \boldsymbol{R}| \,. \tag{5.1}$$

The comparison of this expression with (2.2), (2.3) shows that S_{conv} results from the neglect of the inverse norm kernel in (2.3). It turns out that this neglect may change the results by orders of magnitude (see also below).

The neglect of the norm kernel was of course not explicitly introduced as an approximation, rather it was hidden in another approximation of the reaction theory leading to S_{conv}: The open channel state is of the form $\int d\boldsymbol{R}\, u(\boldsymbol{R})|\boldsymbol{R}\rangle$ with the $|\boldsymbol{R}\rangle$ of (2.1). Since the exact equation for $u(\boldsymbol{R})$ can usually not be solved $u(\boldsymbol{R})$ was taken to be the wave function of relative motion and approximated by the solution of a phenomenological potential (distorted wave approximation). However, $u(\boldsymbol{R})$ is the weight function of the non-normalized basis states $|\boldsymbol{R}\rangle$ and contains therefore necessarily a normalization part $\hat{N}^{1/2}$ (because the complete wave function must be normalized). A distorted wave approximation for $u(\boldsymbol{R})$ neglects by its construction this normalization part.

The neglect of $\hat{N}^{-1}$ in (2.3) leads to an underestimate of the preformation probability by a factor S/S_{conv}. We calculated also S_{conv} for the decay of Ra and obtained the following typical ratios:

$$S/S_{conv} \simeq \begin{cases} 3 \cdot 10^2 & \text{for } \alpha\text{-decay} \\ 4 \cdot 10^8 & \text{for } ^{14}\text{C decay}. \end{cases} \tag{5.2}$$

For α-decay this ratio provided an explanation of the long-standing problem of absolute decay rates [6]). The two numbers in (5.2) show that the norm defects of the states $|\boldsymbol{R}\rangle$ scale similarily as it is expected (4.2) for the spectroscopic factors themselves. We note that a theory for ^{14}C decay based on S_{conv} would be a complete failure: It would yield $S_{conv}/S_{exp} \simeq 10^{-10}$ (to be compared to our result $S/S_{exp} \simeq 10^{-1} \cdots 10^{-2}$).

Appendix

This appendix summarizes the major technical steps for the evaluation of the spectroscopic factor S. In particular the following points are discussed:

- Treatment of the centre-of-mass motion (cmm).
- Reduction of the many-body overlaps.
- Projection onto the nucleon states above the Fermi level.
- Completeness of the basis of gaussian functions.

The SM configurations of sect. 2.2 define the SM states (ν_F: Fermi level, $|0\rangle$: vacuum):

$$|\Phi\rangle = \prod_{\nu < \nu_F} a_\nu^+ |0\rangle . \tag{A.1}$$

The corresponding SM functions $\langle \boldsymbol{r}_i | \Phi \rangle$ contain in contrast to the internal SM functions $\langle \boldsymbol{r}_i | \phi \rangle$ used in sect. 2.1 a spurious cmm of gaussian form. This cmm is

split off from the SM function of the light fragment (^{14}C, α) but (as an approximation) not split off for the heavy ($A \geq 208$) nucleus. This means that the states (2.1) are replaced by

$$\langle \boldsymbol{r}_i | \boldsymbol{R} \rangle = \mathscr{A}\delta(\boldsymbol{R}_{14} - \boldsymbol{R})\phi_{14}\Phi_A , \tag{A.2}$$

where $\boldsymbol{R}_{14}$ is the c.m. coordinate of ^{14}C and $\boldsymbol{r}_i$ are the $A+14$ nucleon coordinates. This approximation has been tested for the α-Pb system; scaling the calculated deviation [10]) for α-Pb with a factor $\frac{14}{4}$ we obtain an estimated error of about 40% for $S(^{14}C)$ due to the replacement (eq. (A.2) instead of (2.1)).

As a next step we introduce a basis of gaussian functions localized at discrete points $\boldsymbol{R}_k$:

$$h_k(\boldsymbol{R}) \propto \exp\left(-\beta(\boldsymbol{R} - \boldsymbol{R}_k)^2\right) . \tag{A.3}$$

For a suitable choice of β the matrix elements of the norm kernel $\hat{N}$ between two such gaussian functions reduce to

$$\langle h_k | \hat{N} | h_{k'} \rangle = \langle \mathscr{A}\Phi_{14}(\boldsymbol{R})\Phi_A | \mathscr{A}\Phi_{14}(\boldsymbol{R}_{k'})\Phi_A \rangle . \tag{A.4}$$

The gaussian functions thus transform $\delta(\boldsymbol{R}_{14} - \boldsymbol{R})\phi_{14}$ (with the internal wave function ϕ_{14}) into the SM function Φ_{14} with a cmm localized at $\boldsymbol{R}$.

The r.h.s. of (A.4) is now an overlap between different Slater determinants of non-orthogonal nucleon states. The most convenient procedere for evaluating this overlap is the following: The nucleon states $|\mu(\boldsymbol{R})\rangle$ ($\mu = 1, \ldots, 14$) of $\Phi_{14}(\boldsymbol{R})$ are orthogonalized (but not normalized) onto the states $|\nu\rangle$ occupied in Φ_A. Then (A.4) is given by the determinant of the overlap matrix $B_{\mu\mu'} = \langle \tilde{\mu}(\boldsymbol{R}_k) | \tilde{\mu}'(\boldsymbol{R}_{k'}) \rangle$ where $|\tilde{\mu}\rangle$ are the orthogonalized states. For the orthogonalization one needs to know the single particle overlaps $\langle \mu(\boldsymbol{R}) | \nu \rangle$ where $|\mu\rangle$ is the $|1s\rangle$ or $|1p, m = 0, \pm 1\rangle$ state localized at $\boldsymbol{R}$ and $|\nu\rangle$ an oscillator state (or different frequency) localized at the origin. For $|1s\rangle$ the analytical expression of this overlap is given in the app. of ref. [18]); from this expression the other overlaps are obtained by a derivative with respect to $\boldsymbol{R}$.

Also the overlap of $\langle \boldsymbol{R} | \Phi_{A+14} \rangle$ with a h_k reduces to an overlap between Slater determinants which can be evaluated in a similar way.

The orthogonalization onto the states $|\nu\rangle$ occupied in Φ_A can be expressed by $|\tilde{\mu}\rangle = \hat{Q}|\mu\rangle$ where

$$\hat{Q} = 1 - \sum_{\nu} c_\nu |\nu\rangle\langle\nu|, \qquad c_\nu = \begin{cases} 1 \\ 0 \end{cases} \text{if } \nu \text{ is } \begin{matrix} \text{occupied} \\ \text{empty} \end{matrix} \text{ in } \Phi_A . \tag{A.5}$$

For partially occupied l-shells the calculation can no longer be reduced to an orthogonalization procedure for each $|\mu\rangle$ state separately. In this case the exact calculation may become rather tedious (in particular for the α-decay of Ra) and has been replaced by choosing effective c_ν's for the corresponding shells. The resulting error is estimated to be limited by about 60% for α and 10% for ^{14}C.

With the described techniques the quantities $\langle \boldsymbol{R}|N|\boldsymbol{R}'\rangle$ and $\langle \boldsymbol{R}|\Phi_{A+14}\rangle$ can be calculated in the space $\{h_k\}$ of gaussian functions. As a last point we discuss the completeness of this space. Due to the non-orthogonality of the h_k the distance ΔR between two neighbouring h_k cannot be made arbitrarily small (otherwise the h_k become numerically linearly dependent rendering the inversion of the overlap matrix $\langle h_k|h_{k'}\rangle$ impossible). Antisymmetrization effects are important if the velocity of the light fragment is smaller than the Fermi velocity or, equivalently, if its momentum is below $14p_F$ (or $4p_F$) where p_F is the Fermi momentum. One expects then that a grid distance ΔR of

$$\pi/\Delta R \sim 14p_F/\hbar\,, \qquad \Delta R \sim 0.17\ \text{fm} \tag{A.6}$$

is sufficient. The actual results for the integral quantity S converge already for $\Delta R \leqslant 0.4$ fm. The numerical error of our calculation due to finiteness of the space of gaussian functions is negligible.

References

1) H.J. Rose and G.A. Jones, Nature **307** (1984) 245
2) W. Kutschera, I. Ahmad, S.G. Armato III, A.M. Friedman, J.E. Gindler, H. Henning, T. Ishii, M. Paul and K.E. Rehm, Phys. Rev. **C33** (1986) 387
3) E. Hourani, M. Hussonnois, L. Stab, L. Brillard, S. Gales and J.P. Schapira, Phys. Lett. **160B** (1985) 375
4) P.B. Price, J.D. Stevenson, S.W. Barwick and H.L. Ravn, Phys. Rev. Lett. **54** (1985) 297
5) A. Sandulescu, D.N. Poenaru and W. Greiner, Sov. J. Part. Nucl. **11** (1980) 528
6) T. Fliessbach and H.J. Mang, Nucl. Phys. **A263** (1976) 75
7) P.R. Christensen and A. Winther, Phys. Lett. **65B** (1976) 19
8) At. Data and Nucl. Data Tables **17** (1976) 88
9) J.B. Ball, C.B. Fulmer, E. E. Gross, M.L. Halbert, D.C. Hensley, C.A. Ludemann, M.J. Saltmarsh and G.R. Satchler, Nucl. Phys. **A252** (1975) 208
10) S. Okabe and T. Fliessbach, Phys. Lett. **144B** (1984) 5
11) T. Fliessbach, S. Okabe, Z. Phys. A **320** (1985) 289
12) Nucl. Data Sheets **22** (1977) 223, 243
13) K.H. Bell, Staatsexamensarbeit, Universität Gesamthochschule Siegen, unpublished
14) I. Perlman and J.O. Rasmussen, in Handbuch der Physik, ed. S. Flügge, vol. XLII, p. 151 (Springer, Berlin, 1957)
15) S. Landowne and C.H. Dasso, Phys. Rev. **C33** (1986) 387
16) Y.J. Shi and W.J. Swiatecki, Nucl. Phys. **A438** (1985) 450
17) T. Fliessbach and P. Manakos, J. Phys. G. **3** (1977) 643
18) T. Fliessbach, Nucl. Phys. **A315** (1979) 109